Mathematical planning in experiments on 'soil-fertilizer-yield' interactions

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A b s t r a c t. The objective of the study was to show the possibility and feasibility of the application of mathematical planning for the determination of the optimum structural parameters and bulk density of the root layer of industrial crops in relation to nutrient supply and other factors. The optimal structural parameters and bulk density were determined for the sowing layers of a typical heavy loam of chernozem type. Patterns of the uptake of nutrients supplied to barley plants were described in relation to agrophysical factors, and to the dosage and application depths of mineral fertilizers.

K e y w o r d s: experimental planning, agrophysical parameters, nutrients

INTRODUCTION

Determination of the optimal agrophysical parameters of the soil root layer of industrial plants is a significant theoretical and practical problem that requires immediate attention.

Investigation of plant requirements as to structural composition and bulk density of the soil in the root layer is one of the major methodological preliminary conditions in the optimization of physical properties and regimes [5]. Requirements of industrial crops can be studied in special experiments using the theory and methods of mathematical planning.

The aim of this study was to show the possibility and feasibility of the application of mathematical planning in the determination of the optimum parameters of soil structure and bulk density in the root layer for different crops as related to nutrient supply level, their covering depth and other factors. At the same time, major patterns of the nutrient uptake by plants as related to the above factors were also described.

OBJECTS AND METHODS

The experiments involved in the study were carried out in the field of the Kommunar Experimental Station of the Institute for Soil Science and Agrochemistry of the Kharkovskiy district, Kharkiv region.

The climate of the experimental location is temperate and continental. The effective temperature sum (above 10) is 2850 C. The average annual air temperature is +6.5 C, the average temperature of the vegetation period is +15.9 C. The average precipitation level measured for several years is 511 mm, and this latter level during the present investigation period was 485-545 mm.

The experiments were conducted on the typical chernozem (Haplic Chernozem) in the Left Bank Forest Steppe of Ukraine. The typical chernozem soil is heavy loam and silty clay. The content of clay particles in the arable layer was 27.1%. Reaction of the soil solution was slightly acidic ($pH_{H_2O} - 5.6$) with hydrolytic acidity of 5.8 mg equiv. The sum of bases absorbed was 35.5 mg equiv. for 100 g of soil. The composition of the absorbed cations was dominated by calcium (78%) and magnesium (20%). The humus content in the arable layer was 3.88%. As concerns the amount of the nutrients available to plants, the soil belongs to a soil group with a low content of soluble phosphorus ($P_2O_5 - 5.2$ mg 100 g of soil) and a medium content of the exchange potassium ($K_2O - 7.8$ mg 100 g of soil) in extract N 0.5 CH₃COOH.

Some special experiments were carried out, both in the laboratory and under field conditions.

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Laboratory experiments

The aim of the investigation was to determine the soil structure in order to optimize the quantitative composition and correlation of structural components which form the sowing layer. Vessels of 5 dm³ and 20 cm high were used. They were filled with soil from the arable layer of the experimental field. A simple 4-component model of structural composition, i.e., crumbs of 20-5 mm (X1), structural macroaggregates of 5-2 mm (X_2) , 2-0.25 mm (X_3) , and silt (less than 0.25 mm $- X_4$) was chosen for the chernozem investigated. Hence, the study on the influence of soil structural composition on crop yield was reduced to a study on a 4-component set of properties (relation of a type called composition – property). Methods of planning, processing and presentation of the experimental results were prepared to investigate these relations. Calculations were conducted in a special mixture simplex coordinate system, where the major correlation was fulfilled, i.e., the sum of all mixture components was 1. A 4-component mixture simplex is a tetrahedron with its height equal to one. In the planes parallel to the simplex sides, one of the components is a constant equal to the distance from the corresponding side. At the opposite apex, the value of the component is equal to 1. Hence, pure components correspond to simplex apexes, double mixtures to edges, triple mixtures to sides, 4-component mixtures to the internal points of the simplex. The results are shown on a tetrahedron formed by a section along the edges from any apex.

The latticed planes of the simplex [8] have two essential shortcomings. Firstly, the properties of multicomponent mixtures are determined by the interpolation of the properties of pure components observed, and of their double mixtures. Secondly, the general relation in the composition-property type can be very difficult (for example, in the presence of phase transition) for an adequate representation by means of common analytical expressions, which can not be allowed in the investigation of soil structural composition. For that reason, the simplex foundation, i.e., lattice planes were modified and became free from the shortcomings mentioned above. The experimental results were processed according to a piece-quadratic model composed of several quadratic pieces for different areas of component changes.

Field experiments

The search for the optimal parameters of soil bulk density with respect to nutrient supply requires multi-factorial schemes of the second order as the minimum, as they allow to determine parameters of the complete quadratic model [3]. For the three factors investigated, plan B₃ was the most economical. It is a selection of 15 variants from CFE³ (complete factorial experiment). For the four-factor scheme, plan B₄ and SFE³⁻¹ (shortened factorial experiment) was the best (24-27 variants in three repetitions). Schemes of the second order allowed the estimation of parameters of the complete quadratic model and, when it was adequate, to find the optimum zones of the parameters investigated. The application of some of them for the experimental setting is discussed below. In one of the experiments on the optimization of the arable layer bulk density for various crops, the application of mineral nutrition further to scheme B_3 was used (Table 1).

Scheme SFE⁴⁻¹ was used (Table 2) in the micro-plot experiment studying the influence of soil bulk density, soil supply with mineral nutrients, application of mineral fertilizers before sowing, and depth of plant cover on crop growth and development yield.

The program for processing multifactor data was elaborated with our assistance. It should become a standard program for multiple regression analysis. To calculate the significance of regression as a whole, the Fisher criterion is calculated as the significance of each of the coefficients (the Student criterion). According to the regression equation, parameters of the optimum zones were determined together with the isoquants (two-parameter diagrams) of the optimization parameters for 2 factors at fixed values; diagrams showing the relations between the optimization parameters for each factor were prepared.

The experiments on the optimization of arable layer bulk density were conducted in field conditions on plots of $1 \times 1 \times 0.3$ m, isolated along a vertical line by means of plastic film. In the experiments carried out during vegetation, the dynamics of the factors selected as well as changes in the optimization parameters under the influence of the factors investigated were studied.

In the above experiments, standard methods approved in Ukraine were used [7]. Root system indices were used according to the method elaborated by Lyndina [4].

In the description of the micro-plot experiments and discussion of results, the following terms were used: sowing layer – a soil layer 4-10 cm thick in which seeds were placed in the majority of field cultures; root layer – a soil layer 0-30 (50) cm thick in which there was the main mass of the root systems of field plants; arable layer – a 0-30 cm thick soil layer, tilled at primary tillage using standard technology.

RESULTS AND DISCUSSION

The optimization of soil structure (results of vegetation experiment)

The phenological investigation revealed that in the first stages of barley development, better growth was observed in the soil with predominating 5-2 mm fractions. Plant development in dispersed soil (domination of fractions of less than 0.25 mm) was slower. It should be added that some soil fertility indices were analytically studied before sowing. Bulk density in various variants was slightly changed (from 1.0 to 1.1 g cm⁻³), total porosity was 61-56 %. There were no essential differences in biological activity (quantity of CO_2 emitted from the soil during 2 h observation was 203-220 g m⁻²), either. The content of main nutrients in mg 100 g⁻¹ of soil follows in the respective variants was as follows: lightly hydrolyzed nitrogen – 24-30: soluble phosphorus – 20-22: exchange potassium – 31-38 (which was due to the fertilizers applied).

At the end of the experiment, bulk density in the experimental variants increased by $0.1-0.2 \text{ g cm}^{-3}$, which corresponded to the decreased porosity; CO2 emitted in the middle and at the end of the experiment increased considerably in comparison with the initial value (458-520 g m⁻²). The soil structural composition after harvesting changed (as compared to the initial status). It took place at the expense of the fractions >5 mm and <0.25. If we consider the initial content of fractions of a certain size as 100%, their finite content in a month's time was as follows for the respective fractions: 20-5 mm - 47%; 5-2 mm - 54%; 2-0.25 mm -66%, <0.25 - 64%. Three months later, at the end of the experiment, the best structural preservation was observed for the medium and large fractions - 5-2 and 20-5 mm (60-48%), and slightly less for the small macro-structural and silt fractions. As a whole, it proved the ability of the soil investigated to preserve its initial structural status. As a result of mathematical processing, an equation of regression describing relation between this last parameter and the factors investigated in the data on crop yield was developed, i.e.:

where: y - dry mass of barley, g vessel⁻¹; $X_1X_2X_3X_4 - content of aggregates, respectively, 20-5. 5-2. 2-0.25 and less than 0.25 mm in the quotas of one. The equation was proved to be true and the multiple correlation coefficient was 0.89. Using the above equation we can calculate barley yield for any component correlation in the mixture. Taking into consideration components of the mixture <math>X_1+X_2+X_3+X_4=1$ (or 100%) used, the area of their change can be described by a regular tetrahedron on an involute for which the maximum correlation of the structural aggregates can be found along the lines connecting equal yield levels.

The optimum ratio of fractions which gives the maximum crop yield is equal to: X_1 -10%, X_2 -20%, X_3 -45% and X_4 -25%. The correlation of structural aggregates determined is tentative, as the minimum zone is essential, since it crosses the tetrahedron from top to bottom (Fig. 1). For the zone mentioned above, domination (to 80%) of structural aggregates less than 5 mm was characteristic. Hence, it can be supposed that in the conditions of lowered moistening and at sufficient supply of nutrients, the arable (sowing) layer should consist mainly of small aggregates.

The same fraction size (less than 5 mm) favours more effective penetration of mineral nutrients into plant roots. The diagram (Fig. 2) obtained is almost analogous to the



Fig. 1. Diagram of barley crop yield dependence (g vessel⁻¹) upon the correlation of the aggregates 20-5 mm (X₁), 5-2 mm (X₂), 2-0.25 mm (X₃) and <0.25 mm (X₄). In lattice knots – the yield experimental data, I – minimum yield zone, II – maximum yield zone.



Fig. 2. Diagram of the dependence of N+P+K entering into plants, (mg vessel⁻¹), upon the correlation the aggregates in the soil structural compaction (X_1, X_2, X_3, X_4). In lattice knots – the experimental data, I – minimum zone of NPK entering into plants, II – maximum zone of NPK entering into plants.

diagram obtained for yield characteristics. It allows for the confirmation of the assumption that the optimum correlation of aggregates determined (at some moisture deficiency) results in a decrease of the coefficient of nutrient usage by plants. Deviation of the structural composition from the optimum correlation structure of fractions promotes a decrease in the efficiency of nutrient usage by plants.

The optimization of the bulk density of arable layer

The possibility of the application of multi-level schemes while searching for the optimum parameters of an arable layer bulk density was examined on the examples of two micro-plot experiments. The scheme of the first one is presented in Table 1. In this experiment, the optimum bulk density (for barley growth) was determined taking into consideration the level of soil mineral nutrition and the covering depth. According to the meteorological conditions, the year of investigation was favourable. The given levels of bulk density were maintained during barley vegetation without considerable changes. According to the data from phenological observations, the best conditions for plant growth and development occurred at an average soil compaction of $1.15 \cdot 1.25 \text{ g cm}^{-3}$. The regression analysis leads to an equation describing changes in barley yield in relation to the factors studied as follows:

$$y=70.9+178.17X_{1}+0.18X_{2}-0.11X_{3}-72.8(X_{1})^{2}-0.19(X_{2})^{2}+ 0.9(X_{3})^{2}+0.99(X_{1}X_{2})-14(X_{1}X_{3})+0.17(X_{2}X_{3}).$$
(2)

The multiple correlation coefficient for the above equation is 0.96. It proves that the model is adequate for the actual data and true.

The following preliminary conclusions can be drawn from the regression equation. Bulk density influences yield formation most of all, less influence is exerted by fertilizer doses. The most insignificant influence is that of fertilizer application depth. For all the above factors, the optimum parameters were determined at the highest crop yield (4.2 t ha^{-1}) . It was proved by the reliable values of their quadratic effects.

A positive correlation mark for bulk density and fertilizers dose $(X_1 X_2)$ showed that bulk density positively

T a b l e 1. The scheme of the micro-plot exp	eriment
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Factors	Levels of factors variation		
rators	0	1	2
X_1 – arable layer bulk density (g cm ⁻³)	0.9-1.0	1.1-1.2	1.3-1.4
X_2 – nitrogen fertilizers doze (kg ha ⁻¹)	30	60	90
X ₃ – application depth (cm)	0-10	10-20	20-30

influences yield formation in a wide range of variation, under the condition of normal supply of mineral nutrition of plants. A negative correlation mark for bulk density and depth of the nitrogen covering $(X_1 X_3)$ showed that the maximum influence of bulk density on the yield formation occurred at a small depth of nitrogen covering, and the minimum - when the covering with fertilizers was at the maximum depth (20-30 cm). A positive mark of the mineral fertilizer factor and covering depth factor (X_2, X_3) showed that small doses of fertilizers should form a cover to a small depth (in our experiment, a nitrogen dose of 90 kg ha⁻¹ corresponded to an average dose recommended for barley grown in the Forest Steppe). Equation series of isoquants can be prepared accordingly (diagrams of relations between parameters with two factors at a fixed value of the third). When requested to fix factors on a certain set level (zero, middle, or high), the computer program will print 9 series of isoquants. However, isoquants can be obtained at any fixed value of each factor. Now, let us examine a plot of isoquant series for grain mass. The isoquants were calculated according to two first factors (bulk density and a dose of nitrogen fertilizer) at a fixed value of the third factor, i.e., depth of covering with nitrogen fertilizer (Fig. 3). The resultant plot demonstrates that the optimum bulk density for barley yield falls into a wide range of values $(1.10-1.32 \text{ g cm}^{-3})$. With deviation of bulk density parameters towards a reduction of grain crop yield (even when doses of nitrogen fertilizers were sufficient, the maximum value of grain yield was observed at a nitrogen dose of 40-80 kg ha^{-1}), it decreased considerably. The minimum crop yield (3.12 tha^{-1}) was observed for loosened soil with low nitrogen dosage. At bulk density increasing above the upper optimum level, crop reduction was also observed. However, as can be seen from the plot of barley grain yield, it is reduced less than for the loosened soil. The same results were obtained in some earlier studies carried out on the same soil [4,5].

Bulk density considerably influences the development of the root system. For the above reason, it is advisable to consider the plant root system as the main criterion for soil bulk density optimization. The mathematical procedure presents data on the development of root system in relation to the factors studied, as described by the following equation:

$$y=4.18-5.88X_{1}-0.4X_{2}-0.09X_{3}+2.37(X_{1})^{2}+0.0025(X_{2})^{2}-0.027(X_{3})^{2}+0.25(X_{1}X_{2})-0.7(X_{1}X_{3})+0.038(X_{2}X_{3}), (3)$$

where: y - root system mass in a 0-60 cm soil layer, t ha⁻¹: X_1 - soil bulk density, g cm⁻³; X_2 - nitrogen fertilizer dose, kg ha⁻¹; X_3 - depth of nitrogen application to the arable soil layer, cm.

Coefficients for the above equation are as follows: multiple correlation (R) – 0.96; determination coefficient (R²) – 0.93. Bulk density influences root growth most of all. A positive nitrogen influence on the root growth (the absolute root mass in the 0-60 cm soil layer) was evident in the condition of depth optimization for fertilizer application to the soil. Figure 4 clearly shows that actually a change of nitrogen fertilizer dose from the middle to the upper level did not influence fertilizer accumulation in the roots of the 0-60 cm layer. The maximum root system mass (7.62 t ha⁻¹) in the present experiment was observed in the loosened arable layer without nitrogen fertilizers. With nitrogen application, the root system mass is reduced. It is true for all crops grown in the conditions of nitrogen nutrition deficiency and can be explained by a possible carbohydrate surplus [1,2].



Fig. 3. The influence of the bulk density (X_1) and nitrogen fertilizers dozes (X_2) at optimum depth of nitrogen application $(X_3=15-20 \text{ cm})$ on barley grain yield (t ha⁻¹).



Fig. 4. The influence of soil bulk density (X_1) and nitrogen fertilizers doze (X_2) at optimum depth of nitrogen application $(X_3=15-20 \text{ cm})$ on the barley root system growth in the arable layer (t ha⁻¹).

Optimisation of the mineral fertilizer application depth in the arable layer

Let us now examine the results of one more micro-plot experiment set according to the SFE^{4-1} scheme (a Latin cube). In this experiment, soil bulk density was optimized in relation to the initial soil supply with nutrients from mineral fertilizers at sowing and their application depth. The experimental scheme is presented in Table 2. In the above experiment, the optimum bulk density (as observed by barley reaction) was estimated with respect to the initial soil supply with nutrients (NPK(0), NPK(400), NPK(1200) applied to the soil 8 years before the experiment); doses of mineral fertilizers at sowing (N₄₀P₄₀K₃₀; N₈₀P₈₀K₆₀; $N_{120}P_{120}K_{90}$) and their covering depth (0-10; 0-20; 0-30 cm). There is a linear relation between conditional and real variable quantities with respect to factors X₁, X₂, X₃ and a non-linear relation with respect to factor X₃; bulk density = $0.95+0.2X_3$; fertilizers doses at sowing = $N_{40}P_{40}K_{30}$ + $(N_{40}P_{40}K_{30})X_2$; initial soil supply with NPK = 400X_31.585; covering depth of pre-sown fertilizers = $10+10X_4$.

Mathematical processing of data on barley data resulted in the following equation of regression:

$$y = 15.2 + 1.32X_1 + 1.75X_2 + 0.82X_3 - 0.87(X_1)^2 - 1.71(X_2)^2 - 0.53(X_2X_3) + 0.43X_2X_4,$$
(4)

where: y - barley grain yield, $t ha^{-1}$; $X_1 - soil bulk density$, $g cm^{-3}$, $X_2 - pre$ -sowing fertilizer dose (NPK), kg ha⁻¹; $X_3 - initial soil supply with nutrients (NPK), kg ha^{-1}$; $X_4 - covering depth of pre-sown NPK, cm.$

The multiple correlation coefficient (R) for the model is 0.91, determination coefficient (R^2) – 0.82. The above proves that the model is adequate for the actual experimental data and shows a considerable impact of the factors studied on barley yield. The yield depended on the experimental factors in 82%.

The following preliminary conclusions can be drawn from the regression equation: the optimum value of X_1 was not determined; a higher bulk density (within the interval for this index in the experiment) meant a higher crop yield. It should be mentioned that the present experiment was

T a b l e 2. Factors and levels of their variations in the micro-plot experiment on the influence of soil bulk density, the initial level of nutrients, the pre-sowing doze of mineral fertilizers and the depth of their covering on the barley yield

Factors —	Level of variations		
	0	1	2
X_1 - soil bulk density in 0-30 cm layer (g cm ⁻³)	0.95-1.05	1.15-1.25	1.35-1.45
X_2 - doze of mineral fertilizers, applied at sowing (kg ha ⁻¹)	$N_{40}P_{40}K_{30}$	$N_{80}P_{80}K_{60}$	$N_{120}P_{120}K_{90}$
X_3 - initial soil provision with nutrients (the sum of the			
applied mineral fertilizers applied formerly) (kg ha ⁻¹)	NPK(0)	NPK(400)	NPK(1200)
X ₄ - fertilizers covering depth at sowing (cm)	0-10	0-20	0-30

conducted during a droughty vegetation period. The optimum value for X₂ was found. The optimum NPK dose for the experimental conditions was at a $N_{90}P_{100}K_{70}$ kg ha⁻¹ level. The optimum value for X3 was determined - a higher initial level of soil supply with fertilizer resulted in a higher crop yield. The sign for the fertilizer interaction when applied at sowing and the pre-sowing soil supply with nutrients was negative. It means that the efficiency of mineral fertilization applied to the soil with low initial level of mineral nutrients was the highest. When the initial soil supply of nutrients was high, the efficiency of mineral fertilization applied at sowing was the minimum. The sign for the interaction of X2 and X3 factors (mineral fertilizer doses and their application depth) was positive. It shows that it is advisable to apply low fertilizer doses at a lower depth when the initial supply with nutrients is high, and to increase these doses as its level gets higher. The maximum experimental yield (5.05 t ha⁻¹) was obtained at the following combination of factors: X_1 -1.32 g cm⁻³; X_2 -N₉₀P₉₀K₆₀; X_3 -25 cm; X_4 -1200 kg ha⁻¹.

Figure 5 shows the optimum values of yield isoquants according to the first X_1 and X_2 at X_3 and X_4 factors. The maximum yield zone was determined at 1.17-1.40 g cm⁻³ of the bulk density (theoretical state of optimum is 1.32 g cm^{-3}). The optimum NPK dose was $N_{70}P_{70}K_{40}$ as the maximum barley yield was observed in the range of $N_{70}P_{70}K_{90} - N_{120}P_{120}K_{80}$. A further increase of mineral fertilizer dose in the experimental conditions decreased the yield (the effect of moisture deficiency). Figure 6 presents yield isoquants for the same two factors (bulk density and presowing fertilizer dose). The third factor, i.e., initial nutrient supply was fixed at the maximum level, and the fourth –

depth of fertilizer application at sowing – was fixed at the minimum level of 10 cm. The above plot supports the conclusion that the influence of the depth of mineral fertilizer application on yield was weak as compared to other factors, as predicted from the regression equation. On the basis of the above plots the following conclusion can be drawn: optimization of depth of mineral fertilizer application increases mineral fertilizer efficiency. High doses of fertilizers should be applied deeper than 10 cm (15-30 cm).

CONCLUSIONS

1. Investigations of the 'soil-fertilizer-yield' system in order to determine the optimum level of the factors which provide the necessary parameters should be carried out using the theory and methods of experimental mathematical planning.

2. Experimental schemes which allow for the collection of necessary information with a low number of experimental variants in a number of repetitions are preferable.

3. In experiments on the soil structural composition, the most economical and informative are the schemes with 'composition – properties' (4-component simplex-latticed schemes).

4. To determine interactions in the system with 'bulk density – agrochemical and agrotechnical factors – plants productivity', it is advisable to use three-level schemes. When 3 factors are investigated, the most economical is scheme B_3 which is a selection of variants (15) from CFE⁻³3 and uses triple repetitions of a minimum number of variants. For the four-factor experiment, the most advisable is B_4 scheme (24 variants).



Fig. 5. The influence of the soil bulk density (X_1) and mineral fertilizers pre-sowing doze (X_2) on barley crop yield (t ha⁻¹). The soil initial provision with nutrients $(X_3) - (NPK)$ 1200, the application depth of the pre-sowing doze $(X_4) - 25$ cm.



Fig. 6. The influence of the soil bulk density (X_1) and mineral fertilizers pre-sowing doze (X_2) on barley crop yield, t ha⁻¹. The soil initial provision with nutrients $(X_3) - NPK(1200)$, the application depth of the pre-sowing doze $(X_4) - 10$ cm.

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