AccurACY OF SOIL MOISTURE MEASUREMENT BY TDR TECHNIQUE

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Accepted January 25, 2000

Abstract. The error in the measurement of volumetric water content by the TDR technique results from the correlation imperfections between the directly measured values of soil refractive index, \( n \), and the real value of soil moisture as well as the hardware and software imperfections of the TDR device and TDR probe installation. On the base of the laboratory measurements of the selected mineral, organic soils and their mixtures, it was confirmed that the soil solid phase significantly influences moisture values as determined by the TDR. Inclusion of the soil bulk density in the TDR calibration formula decreases the absolute error of the TDR determined soil moisture by the factor of two. The relative error of TDR moisture values increases in the lower range of water contents. This is due to a constant absolute error introduced by the measuring device and an increasing role of the soil solid phase in the soil refractive index.

Keywords: time domain reflectometry, soil moisture, measurement error

INTRODUCTION

The Time Domain Reflectometry (TDR) method for the measurement of soil water content becomes more and more popular for its simplicity of operation, accuracy and non-destructiveness, as compared to other methods [5,6]. This measuring technique takes advantage of three physical phenomena characteristic to the soil, i.e.: 

(i) in the frequency range of 1 GHz the complex dielectric constant of the soil is reduced to its real value and the electromagnetic wave propagation velocity, \( v \), in the soil can be expressed as:

\[
v = \frac{c}{\sqrt{\varepsilon(\theta)}} = \frac{1}{n} = \frac{2 \cdot L}{\Delta t}
\]  

where: \( c \) is velocity of light in free space, \( \varepsilon(\theta) \) is a real part of the complex dielectric constant related to its moisture, \( \theta \); \( n = \sqrt{\varepsilon(\theta)} \) is the soil refractive index; \( L \) is the length of the TDR probing rods inserted into the soil; \( \Delta t \) is the time distance between reflections of the TDR pulse from the beginning and the end of the probing rods, inserted into the soil [7];

(ii) a dielectric constant of the soil liquid phase has much higher value than other soil phases, i.e., about 80 against 2 for the solid and 1 for the gas phase;

(iii) the relation between the soil moisture and its dielectric constant is highly correlated for the majority of soils [7,11].

As in indirect method of measurement, the TDR measured moisture values are burdened with correlation imperfections between the measured values, i.e., soil refractive index \( n = \sqrt{\varepsilon} \) and the actual value of the soil moisture, \( \theta \). It is assumed that the “real value” of soil moisture is its value measured by the reference thermogravimetric method. The other sources of errors are: hardware and software imperfections of the measuring device (which measures the time distance between the reflections of the electromagnetic pulse from the beginning
and the end of the TDR probe rods [7]) and TDR probe installation.

The user of the TDR meter has limited possibilities to minimize the influence of the hardware and software errors. He can only perform repeated measurements from the same probe and take the mean of the received data for further analysis. Care should be taken during TDR probe installation, so as to avoid air gaps [1], stones and roots between steel rods and the soil. The picture showing the reflections from the beginning and end of the probe inserted into the soil would be helpful. With some experience, the user can decide, if the measurement point is representative or not.

For the discussion about the correlation error, it is useful to treat the soil as a three phase medium, where the following formula applies [8]:

\[ \frac{1}{\varepsilon} = \frac{1}{\varepsilon_w} + \frac{1}{\varepsilon_s} + \frac{1}{\varepsilon_a} \]

\[ n = \frac{\rho_s}{\rho} (n_s - 1) + \theta (n_w - 1) + 1 \]  

\[ n = \sqrt{\varepsilon} = n_s (1 - \phi) + n_w \theta + \phi - \theta \]

where \( n = \sqrt{\varepsilon} \) is a soil refractive index, while \( n_s \) and \( n_w \) are respectively the refractive index of the solid phase and water of the soil (we assume that \( \sqrt{\varepsilon_a} = n_a = 1 \)).

The Eqs (5) and (6) show that apart from the liquid phase, the solid phase influences dielectric properties of the soil. The statistical significance of this influence and the resulting error of the TDR measured soil refractive index and the soil moisture derived from it are discussed below.

MATERIAL AND METHOD

The measurements were performed on disturbed mineral and organic soil samples in laboratory conditions. The measured material consisted of: 395 soil samples of 19 mineral soils different in texture, organic carbon content (C), bulk density (\( \rho \)), and particle density (\( \rho_s \)), 111 soil samples of 9 organic soils different in organic carbon content (C), bulk density (\( \rho \)), and particle density (\( \rho_s \)), 157 soil samples of 9 soil mixtures of peat-silt and peat-sand with differentiated bulk density (\( \rho_s \)), and particle density (\( \rho_s \)). A detailed description of physical parameters of the soil used in the experiment is presented in [9]. Each of the soil sample had its moisture, \( \theta \), determined by the standard thermogravimetric method and the soil refractive index, \( n \), was measured by the instruments produced by the EASY TEST, Ltd. from Lublin, Poland on the license of the Institute of Agrophysics, Polish Academy of Sciences, Lublin [4].

RESULTS AND DISCUSSION

The correlation table calculated for the selected soil parameters is presented in Table 1. The soil volumetric water content, \( \theta \), is highly correlated with the refractive index, \( n \), for the analysed set of soil samples. The other soil physical parameters: \( \phi, \rho_s, \rho \) and \( C \) are not
correlated with \( n \). The soil texture represented by the clay content in the soil (clay), analysed only for the mineral soils (395 cases), is not correlated with \( n \), either. High correlation between \( \theta \) and \( n \) \((R^2=0.92)\) enables to estimate the soil moisture in the indirect way by the measurement of the soil refractive index, \( n \). The relation between \( n \) and \( \theta \) was calculated by fitting a regression line into the set of measured data \((n,\theta)\) is:

\[
n(\theta) = a_0 + a_1 \theta
\]

where \( n(\theta) \) is the soil refractive index determined by TDR method, \( \theta \) is the moisture of the tested soil samples determined by thermogravimetric method, \( a_0 \) and \( a_1 \) are: offset and slope of the trend line fitted into the data pairs \((n,\theta)\).

The formula (7), called as the TDR calibration line, was determined for the whole set of data (663 soil samples) and separately for each subset of data (Table 2).

The values of the offset, \( a_0 \), and the slope, \( a_1 \), are different for the selected soil types; both values are the smallest for the organic soils and the highest for the mineral soils. Statistical parameters \( R^2 \) and \( S_{yx} \) (standard error of estimation) calculated for all the 663 soil samples indicate worse correlation than for the individual soil types. The values of \( R^2 \) for the mineral and organic soils have similar values, nearly as high as for soil mixtures (Fig. 1). Lower \( R^2 \) value of and higher \( S_{yx} \) value of for the soil mixtures correspond with great differentiation of their density and porosity. Therefore it can be concluded that apart from the soil moisture, \( \theta \), also soil density (or porosity) influences soil refractive index, \( n \), measured by TDR method.

The measured data were tested by the multiple regression method to determine the influence of the analysed soil properties on the value of the refractive index, \( n \):

<table>
<thead>
<tr>
<th>Soil</th>
<th>No. of cases</th>
<th>Density (g cm(^{-3}))</th>
<th>Porosity</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( S_{yx} )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mineral</td>
<td>395</td>
<td>1.01±1.80</td>
<td>0.32±0.65</td>
<td>1.421</td>
<td>8.976</td>
<td>0.219</td>
<td>0.977</td>
</tr>
<tr>
<td>Peat-sand and peat-silt mixtures</td>
<td>157</td>
<td>0.59±1.43</td>
<td>0.46±0.74</td>
<td>1.200</td>
<td>8.008</td>
<td>0.217</td>
<td>0.976</td>
</tr>
<tr>
<td>Organic</td>
<td>111</td>
<td>0.12±0.65</td>
<td>0.53±0.91</td>
<td>0.776</td>
<td>7.932</td>
<td>0.260</td>
<td>0.981</td>
</tr>
<tr>
<td>All soil samples</td>
<td>663</td>
<td>0.12±1.80</td>
<td>0.32±0.91</td>
<td>1.491</td>
<td>7.675</td>
<td>0.425</td>
<td>0.923</td>
</tr>
</tbody>
</table>

Table 1. The squared values of the correlation coefficient, \( R^2 \), between the selected parameters of the analysed set of data, No. of cases 663

Table 2. Regression parameters \( n(\theta) \) for the tested soils (\( R^2 \) - squared value of the correlation coefficient, \( S_{yx} \) - standard error of estimation)
\[ n = a_0 + a_1 \theta + a_2 \rho + a_3 \phi + a_4 \rho_s + a_5 C + a_6 \text{clay} \]  

(8)

where: \(a_0\) to \(a_6\) are coefficients connected with different soil parameters: \(\theta\) - moisture determined by the reference thermogravimetric method, \(\rho\) - bulk density, \(\phi\) - porosity, \(\rho_s\) - particle density, \(C\) - organic carbon content, \(\text{clay}\) - clay content in the analysed samples.

The analysis of the \(\text{clay}\) significance in the Eq. (8) was performed only for the mineral soils (395 cases). Table 3 presents the values of the coefficients \(a_0\) to \(a_6\) and the parameters related to the t-test (\(p_t\)) and F-test (\(p\text{-value}\)) for all the soil samples. Each individual combination from the first column refers to the regression function (8) with arbitrarily chosen number of parameters \(a_i\).

For each combination of independent data (i.e., \(\theta\), \(\rho\), \(\rho_s\), \(C\), and \(\text{clay}\)) the \(p\)-values of respective \(a_i\) coefficients are calculated. The assumed significance level is 0.001. Except \(a_5\) in the third combination all the coefficients have their \(p\)-values below 0.001, which proves that they are statistically significant. Therefore the inclusion of \(\rho_s\) or \(C\) and \(\text{clay}\) in the regression equation improves its parameters \(R^2\) and \(S_{yx}\). This proves that the properties of soil solid phase influence the value of the soil refractive index, \(n\), and

**Table 3.** The results of multiple regression analysis for all analysed soil samples (No. of cases - 663)

<table>
<thead>
<tr>
<th>No.</th>
<th>Regression parameter</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2) ((\theta))</th>
<th>(a_3) ((\rho))</th>
<th>(a_4) ((\rho_s))</th>
<th>(a_5) ((C))</th>
<th>(p\text{-value})</th>
<th>(R^2)</th>
<th>(S_{yx})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a_i)</td>
<td>1.491</td>
<td>7.675</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.923</td>
<td>0.425</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(p_t)</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a_i)</td>
<td>0.289</td>
<td>8.504</td>
<td>0.846</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.979</td>
<td>0.220</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(p_t)</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>(a_i)</td>
<td>-0.279</td>
<td>8.500</td>
<td>0.792</td>
<td>0.244</td>
<td>0.005</td>
<td>&lt;0.001</td>
<td>0.980</td>
<td>0.218</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(p_t)</td>
<td>0.092</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>0.005</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>(a_i)</td>
<td>2.610</td>
<td>8.436</td>
<td>-2.463</td>
<td>-</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>0.973</td>
<td>0.253</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(p_t)</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>(a_i)</td>
<td>2.335</td>
<td>8.491</td>
<td>-1.857</td>
<td>-</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>0.976</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(p_t)</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(a_i)</td>
<td>1.491</td>
<td>8.304</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.962</td>
<td>0.299</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(p_t)</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
consequently the velocity of EM wave propagation in the soil.

The values of p (referring to the F-Snedecor test) do not exceed 0.001 in any presented combinations. This does not agree with the zero hypotheses claiming that all the coefficients in the analysed multiple regression equation have zero values. Therefore each analysed regression model is statistically significant. The reason for it may be a high correlation between the variables and . The inclusion of additional variables into the regression Eq. (8) (i.e., introducing more degrees of freedom), apart of the soil moisture, can only decrease the value of p parameter. The lower p, the goodness of fit in multiple regression is better.

Table 4 presents the results of a multiple regression analysis for the individual groups of soil. Selection of data into three individual groups (mineral, organic soils and their mixtures) significantly increases the correlation coefficient, R, and decreases the standard error of estimation, , between and n in the linear relation in all the groups as compared to all data. Introduction of the soil solid phase parameters into the multiple regression Eq. (8), i.e., its density, , or porosity, , additionally increased R and decreased . For each combination of independent variables the regression is statistically significant because p-values calculated for the respective cases are always lower than the assumed error level (equal to 0.001).

The analysis of the significance of soil texture in the regression Eq. (8), represented by the clay content was performed in the group of mineral soils. It was found out that the soil clay content significantly influences the regression equation coefficients (Table 4, case No. 2 in the group of mineral soils). Importance of the soil clay content as an element in the regression is almost the same as the soil density. The regression yielded and the same values as for regression.

<table>
<thead>
<tr>
<th>No.</th>
<th>Regression parameter</th>
<th>a0</th>
<th>a1 (θ)</th>
<th>a2 (ρ)</th>
<th>a3 (φ)</th>
<th>a4 (ρc)</th>
<th>a5 (C)</th>
<th>a6 (clay)</th>
<th>p-value</th>
<th>R²</th>
<th>Sᵧx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a1</td>
<td>1.422</td>
<td>8.976</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.977</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
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</tr>
<tr>
<td>2</td>
<td>a1</td>
<td>1.502</td>
<td>9.134</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.004</td>
<td>&lt;0.001</td>
<td>0.981</td>
<td>0.198</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a1</td>
<td>0.604</td>
<td>9.014</td>
<td>0.561</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.982</td>
<td>0.191</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
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<tr>
<td></td>
<td>Mineral soils (395 cases)</td>
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<tr>
<td></td>
<td>a1</td>
<td>0.776</td>
<td>7.932</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>&lt;0.001</td>
<td>0.981</td>
<td>0.260</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>2</td>
<td>a1</td>
<td>0.844</td>
<td>7.951</td>
<td>0.238</td>
<td>-0.171</td>
<td>0.002</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.982</td>
<td>0.255</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>0.003</td>
<td>&lt;0.001</td>
<td>0.173</td>
<td>0.152</td>
<td>0.461</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>3</td>
<td>a1</td>
<td>0.876</td>
<td>7.946</td>
<td>-0.370</td>
<td>-0.004</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.982</td>
<td>0.256</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>0.116</td>
<td>0.052</td>
<td>-</td>
<td>-</td>
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<tr>
<td></td>
<td>Organic soils (111 cases)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>a1</td>
<td>1.200</td>
<td>8.008</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.976</td>
<td>0.217</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a1</td>
<td>2.573</td>
<td>8.142</td>
<td>1.097</td>
<td>0.912</td>
<td>0.073</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.994</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>3</td>
<td>a1</td>
<td>3.448</td>
<td>8.190</td>
<td>-4.399</td>
<td>-0.606</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>&lt;0.001</td>
<td>0.993</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>p₁</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Soil mixture (157 cases)</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Taking into account regression analysis with all the analysed soil samples, it can be found that apart from the primary influence of the soil moisture, \( \theta \), on the value of the soil refractive index, \( n \), also the soil solid phase modifies the relation \( n(\theta) \) by the soil density, \( \rho \), and \( \rho_s \) (or the soil porosity, \( \phi \)) in the statistically significant way.

On the basis of the performed multiple regression analysis the following empirical formulae were derived:

**Model I (Table 3, case No. 1):**

\[
n_1 = 1.491 + 7.675\theta. \quad (9)
\]

**Model II (Table 3, case No. 2):**

\[
n_{II} = 0.289 + 8.504\theta + 0.846\rho. \quad (10)
\]

Taking into account the significance of the soil density in the \( n(\theta) \) relation, the following modification of the Eq. (7) is proposed:

\[
n = (a_1 + a_2\rho) + (b_1 + b_2\rho)\theta \quad (11)
\]

where the offset \((a_1 + a_2\rho)\) and the slope \((b_1 + b_2\rho)\) of the \( n(\theta) \) line are linear function of soil density.

The parameters \( a_1, a_2, b_1 \) and \( b_2 \) in the Eq. (11) were found with the use of the least square method giving the formula:

\[
n_{III} = 0.573 + 0.582\rho + (7.755 + 0.792\rho)\theta \quad (12)
\]

referred to as the model III.

On the basis of Eq. (5) and the set of measured data \((n, \rho, \rho_s, \theta)\), using the least square method for all the analysed data (663 soil samples), the values of the EM refractive index of water, \( n_w = 8.676 \), and soil solid state, \( n_s = 2.177 \), were found. Introducing these values into Eq. (6), we arrive at:

\[
n_{IV} = 2.177 + 7.676\theta - 1177\phi \quad (13)
\]

referred to as the model IV.

The water refractive index, \( n_w \), in the temperature of 20 °C, and the frequency of 1 GHz, is 8.95 [3]. The difference between the table and calculated experimental values of \( n_w \) suggests that its value decreases for the soil water. The reason for the above is the fact that water particles are bound by the soil solid phase, which decreases mobility of water dipoles.

Equations (9), (10), (12) and (13) present four models describing soil refractive index, \( n \), in the function of the selected soil properties. These models were verified by the comparison between the model calculated values of \( n \) with the experimental ones measured with the use of Time Domain Reflectometry technique. Figure 2 displays the results of this verification. The values of \( R^2 \) and \( S_{xy} \) and the convergence of the trend line with the 1:1 line is the best for model III. This model generates the \( n \) values, which are the closest to the measured ones. Model I, presently used as the calibration basis [4] in the TDR soil moisture measurement, generates values with the poorest correlation with the measured ones. Models II and IV are in the middle as far as their fit correctness is analysed.

The best of the analysed models is model III described by Eq. (12). Taking into account the soil bulk density, \( \rho \), decreases the standard deviation, \( S_{yx} \), almost twice as much as in the model I, which has only soil moisture, \( \theta \), as the independent variable. Model IV with two independent variables: soil moisture, \( \theta \), and porosity, \( \phi \), generates theoretical values of \( n \) slightly worse correlated with the measured values than model III.

The presented models allow for the determination of soil moisture from the measurement of soil refractive index. The relation \( \theta(n) \), with only one independent variable, can be found by conversion of Eq. (9), i.e., from model I:

\[
\theta_{TDR\_I} = 0.134n - 0.182. \quad (14)
\]

Relation \( \theta(n, \rho) \) can be found by conversion of the Eq. (10), i.e., from the model III:

\[
\theta_{TDR\_III} = \frac{n - 0.573 - 0.582\rho}{7.755 + 0.792\rho} \quad (15)
\]

with two independent variables: \( n \) and \( \rho \). Equation (15) is proposed as the calibration formula of the TDR soil moisture because of the best correlation of data.

Figure 3B presents a comparison of the soil moisture values, \( \theta_{TDR} \), received from model III
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Fig. 2. Comparison of model values of the EM refractive index of the soil with the use of A - model I, B - model II, C - model III and D - model IV, with the values measured with TDR device (horizontal axis).

Fig. 3. Comparison of the values of soil moisture, $\theta_{TDR}$, calculated from the measured values of soil refractive index, $n$, for model I - A and model III - B with the reference values determined by the thermogravimetric method.
with the reference values, \( \theta \), measured by the thermogravimetric method. The slope equal to one, almost no offset of the trend line and low value of the standard error of estimation, \( S_{yx} \), verifies model III as the calibration equation of the reflectometric measurement of volumetric water content. The similar values for model I presented in Fig. 3A show bigger dispersion of results generated by the actually used calibration function Eq. (9).

Equation (16) presents the soil moisture values determined by TDR on the base of model I as the function of directly measured values of time distance between the appropriate reflections from the TDR probe:

\[
\theta_{\text{TDR}} = 0.134 \frac{c_t}{2L} - 0.182. \tag{16}
\]

The derivative of Eq. (16) on time, \( t \), is:

\[
\Delta \theta_{\text{TDR}} = 0.134 \frac{C}{2\Delta t} \tag{17}
\]

where: \( \Delta \theta_{\text{TDR}} = (\theta_{\text{TDR}} - \theta) \) is the absolute error of the TDR measured soil moisture, and \( \Delta t \) is the absolute error of the propagation time measurement.

For the practical TDR sensor length \( L = 10^{-1} \) m, and the absolute error of time measurement \( \Delta t = 30 \cdot 10^{-12} \) (s), the absolute error of soil moisture measurement resulted from the hardware and software imperfections of the TDR meter is:

\[
\Delta \theta_{\text{TDR}} = 0.2 \cdot 10^9 \Delta t = 0.006 = \pm 0.003. \tag{18}
\]

The relative error of the soil moisture measurement, \( \Delta \theta_{\text{TDR}} \), calculated from the measurement of its refractive index, \( n \), and originated from the constant error of the measuring device is:

\[
\Delta \theta_{\text{rel}} = \frac{\Delta \theta}{\theta} = \frac{0.2 \cdot 10^9 \Delta t}{\theta} = 0.006. \tag{19}
\]

Equation (19) presents the value of the relative error of \( \theta \) measurement, which results only from the measuring device. Its value depends on the soil moisture, i.e., it tends to infinity for dry soil and for water it equals 0.006. Introduction of an empirical correction on \( \theta_{\text{TDR}} \) for the solids and gas phases influence can not decrease the value of \( \Delta \theta_{\text{rel}} \) below the value presented in the Eq. (19).

Figure 4 presents the relation between the reflectometric soil moisture measurement error, \( \Delta \theta_{\text{TDR}} \), and its real value, \( \theta \), for models I and III. In the right top corners of these pictures there are the histograms of errors. Smaller dispersion of data points and the shape of histograms confirm the importance of soil density as a factor influencing the error of the TDR soil moisture measurement.
From Table 5 presenting the selected statistical parameters of the absolute errors referring to the models under investigation, it is evident that model III works much better than model I. Inclusion of soil density into model III resulted in the decrease of standard deviation of the TDR absolute error. Also the mean value of this error was closer to zero.

<table>
<thead>
<tr>
<th>Table 5. Statistical parameters of the absolute errors for the models under investigation</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>Mean value</td>
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<tr>
<td>Standard deviation</td>
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<td>Maximal value</td>
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<tr>
<td>Minimal value</td>
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<tr>
<td>Number of cases</td>
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Similar analysis was done for the relative error of TDR soil moisture measurements.

Figure 5 presents relations between $\Delta \theta_{\text{rel}}$ and $\theta$ for both models (in the same scale). Also the histograms of the error $\Delta \theta_{\text{rel}}$ for model I and model III are presented. Ideally, the histograms should look like a normal curve. Higher values of $\theta_{\text{rel}}$ for the lower values of $\theta$ can be explained by bigger influence of the soil solid phase in the effective soil dielectric constant. Water in dry soil exists mainly in the form of particles adsorbed by the solid phase. In this condition, water molecules are not as easy to polarize as in the free water condition. Therefore, the TDR soil moisture measurement should underestimate the real values, as compared to the thermogravimetric method of the soil moisture determination. This effect should be more acute for fine soils. Also the error coming from the hardware, which is constant, is more visible for low water contents, according to Eq. (19). The relative error of the TDR moisture values increases practically to infinity when soil moisture tends to go close to the zero value. Comparison of statistics for the relative errors of the TDR moisture calculated from models I and model III is presented in Table 6. For the soil moistures above 0.1, the standard deviation of $\Delta \theta_{\text{rel}}$ is about four times smaller than for the whole range of $\theta$.

Sensitivity of the TDR moisture on the soil density values calculated according to model III and expressed as $\Delta \theta_{\text{TDR}} / \partial \rho$ on the basis of Eq. (15) is shown in Fig. 6.

Figure 6 shows that neglecting the change of soil density of 0.1 g cm$^{-3}$ results in the TDR moisture determined error of $\Delta \theta_{\text{TDR}} = 0.0064$ for dry soil and $\Delta \theta_{\text{TDR}} = 0.012$ for the soil with the moisture value of 0.5.

---

**Figure 5.** The relative error, $\Delta \theta_{\text{rel}}$, of the TDR soil moisture results as related to the real values of its moisture, $\theta$, and the histogram of $\Delta \theta_{\text{rel}}$ for soil moisture values calculated for model I: A and model III: B.
CONCLUSIONS

Soil solid phase influences the electromagnetic wave velocity (and consequently its dielectric constant) in soil in the statistically significant way. This influence makes the reflectometric technique, in the soil moisture measurement application, not selective. Soil density or porosity represents the solid phase influence on its dielectric constant measured by TDR. Inclusion of this influence in the calibration formula of the TDR method makes the reflectometric method of soil moisture measurements more selective and decreases the absolute error of its measurement by the factor of two.

The error of TDR moisture determination originates mainly from the correlation imperfections. The TDR hardware and software sources of TDR moisture error may be visible in the lower range of soil moisture values.

REFERENCES