WEAR-IN EFFECTS ON LOADS IN CYLINDRICAL AND CONICAL MODEL
GRAIN BINS

J. Horabik¹, M. Molenda¹, I.J. Ross²

¹Institute of Agrophysics, Polish Academy of Sciences, Doświadczalna 4, P.O. Box 201, 20-290 Lublin 27, Poland
²Biosystems and Agricultural Engineering Department, University of Kentucky, 128 Agricultural Engineering Bldg.
Lexington, Kentucky 40546-0276, U.S.A.

Accepted January 19, 1998

Abstract. The vertical wall and floor loads exerted by soft red winter wheat on flat-floor model bins are compared for cylindrical and conical shape bins. The change in the inclination of the bin wall from 0° to 6° with respect to the vertical direction significantly decreased all measured and calculated load parameters.

Keywords: wheat, loads, friction, bin

INTRODUCTION

Bin walls diverging from top to bottom are subjected to reduced vertical loads as compared to vertical bin walls. Moysey and Landine [8] have found that vertical and lateral wall loads in a bin with a wall diverging from top to bottom are significantly less than those in a bin with a vertical wall even for a wall inclination angle as small as 0.73°. Ross et al. [9] have measured the percentage of the total grain load supported by the wall of a model bin with different angles of wall inclination. They observed that a wall inclination angle equal to 6 reduced the percentage of the total grain load supported by the wall by as much as 36% as compared to the vertical wall. Molenda et al. [6] have used the differential slice method based on Janssen’s [4] approach to approximate the pressure distribution in a conical shaped bin. They found a significant effect of the wear-in process on the wall load decrease and flow pattern change. Reduction of vertical wall load in conical shaped bins as compared to vertical wall bins may offer the bin designer an opportunity to use lighter materials for conical shaped storage bins. Comparison of load distributions in conical and vertical wall bins can supply them with information relevant for the design process.

The purpose of this study was to compare the vertical wall and floor loads exerted by wheat on flat-floor cylindrical and conical shape bins.

JANSSEN’S APPROACH

The well-known Janssen [4] formula is widely used to estimate pressures in deep vertical wall bins. The equations predict exponential distributions of vertical and lateral pressure exerted on a bin by a granular material:

\[ \sigma_x = \frac{\gamma D}{4k\mu} \left(1 - e^{\frac{4k\mu z}{D}} \right) \]

where \( D \) - bin diameter, \( k \) - lateral to vertical pressure ratio (Janssen’s constant), \( \sigma_x \) - vertical coordinate (depth of grain in bin), \( \gamma \) - unit weight, \( \mu = \tan \varphi_w \) - coefficient of wall friction, \( \sigma_z \) -

*Paper presented at 6 ICA
lateral pressure, $\sigma_z$ - vertical pressure. The vertical wall load-to-total grain load ratio (VWL/TGL) predicted by Jansen's formula is a function of pressure ratio, $k$, the coefficient of wall friction, $\mu$, and the height to diameter ratio, $H/D$, [3]:

$$VWL/TGL = \left( 1 - \frac{1-e^{-4k\mu \frac{H}{D}}}{4k\mu \frac{H}{D}} \right) \times 100\% \quad (2)$$

where $H$ - height of grain in the bin.

A set of values of the $k$ product can be predicted from the relations between the stress components considering different assumptions of the stress state in the differential layer of the granular material. Assuming for the filling and static situations that an active state of stress is developed in the granular material with yielding at the wall, and the vertical pressure, $\sigma_z$, is constant across the differential slice, the product, $k\mu$, (denoted as a factor $K$) can be expressed as a function of the angle of internal friction, $\delta$, and the angle of wall friction, $\varphi_w$ [1,7]:

$$K = \frac{\sigma_f}{\sigma_z} = \frac{\sin \delta \sin \alpha}{1 + \sin \delta \cos \alpha} \quad (3)$$

where

$$\alpha = \arcsin \left( \frac{\sin \varphi_w}{\sin \delta} \right) - \varphi_w. \quad (4)$$

The same procedure can be applied to analyze stress components in the differential slice in a conical bin. The Mohr diagrams of the stress components for material in the cylindrical and conical bins are shown in Fig. 1a and b, respectively. Using the same assumptions as those for the vertical wall, the $K$ factor can be expressed as follows:

$$K = \frac{\sin \delta \sin \alpha}{1 + \sin \delta \cos (\alpha + 2\theta)} \quad (5)$$

where $\theta$ - angle of wall inclination to the vertical.

The vertical pressure distribution along the height of the conical bin obtained with the differential slice method [1,6] is given by the power function of the vertical coordinate $z$:

$$\sigma_z = \frac{V^2}{P} \left( 1 - \frac{(l/z)^p}{(l/z)^p} \right) \quad (6)$$

where

$$P = 3 - 2K (\cot \varphi_w - \cot \theta) \quad (7)$$

$l$ - distance from the vertex of an imaginary cone forming a conical bin to the free surface of the grain.

The vertical wall load-to-total grain load ratio (VWL/TGL) using Eq. (6) is:

$$VWL/TGL = \left( 1 - \frac{\left( 1 - \frac{(l/H_m)^p}{(l/H_m)^3} \right)}{\left( 1 - \frac{(l/H_m)^p}{(l/H_m)^3} \right)} \right) \times 100\% \quad (8)$$

where $H_m$ - distance from the vertex of an imaginary cone forming a conical bin to the bin floor.

A graph of the $VWL/TGL$ ratio as the function of the angle of wall friction, $\varphi_w$, for the $H/D$ ratio equal to 2.75 in the cylindrical (Eq. 2) and conical (Eq. 8) bins is shown in Fig. 2. The $VWL/TGL$ ratio decreases as the wall inclination angle, $\theta$, increases for any given angle of wall friction, $\varphi_w$. The $VWL/TGL$ ratio is zero for $\varphi_w$ equal to $\theta$. This indicates that the total weight of grain is supported by the bin bottom with no vertical load exerted on the pin wall if the wall angle of inclination, $\theta$, is equal to wall angle of friction, $\varphi_w$. For a vertical wall bin, this situation corresponds to a frictionless contact. For the conical bin, it means that the downward directed vertical component of the wall shear stress, $\sigma_w$, equals to the upward directed vertical component of the pressure normal to wall, $\sigma_n$. The resultant stress vector, $\sigma$, exerted on the conical wall is directed horizontally as shown in Fig. 2. When the angle of wall friction is lower than the wall inclination angle, $\varphi_w < \theta$ the upward directed vertical component of the normal to wall pressure, $\sigma_n$, is larger than the downward directed vertical component of the wall shear stress, $\sigma_w$. 


component of wall shear stress, $\sigma_r$, is greater than the upward directed vertical component of the normal to wall pressure, $\sigma_n$, and the $VWL/TGL$ ratio is positive.

EQUIPMENT AND PROCEDURES

Experiments were performed on cylindrical and conical shape smooth-wall model bins equipped with flat bottoms. Both bins were 7.3 m high and 2.44 m in diameter at the base. The conical bin had an angle of the wall inclination with respect to the vertical direction equal to 6. The flat floor of both bins and the walls were each supported independently on three load cells to isolate wall and bottom loads. Load cells were spaced at an angular distance of 120 around the circumference of the bin. A schematic diagram of the two model bins with the locations of the load cells are shown in Fig. 3.

The bins were centrally filled from a spout that discharged at the top of the bins and centrally discharged. The orifice was 8.9 cm in diameter which produced a sliding velocity of wheat during plug flow of 3.1 m/h along the vertical wall. After filling, the test bins were allowed to equilibrate during a holding period.
of 0.5 h. The loads during filling, detention and discharge were measured at one minute intervals until discharge was completed. The loads were measured with an accuracy of ±50 N. Soft red winter wheat with an initial uncompacted bulk density of 750 kg/m$^3$ and a moisture content of 11.5% (w.b.) was used for tests on both bins. All experiments were conducted in a temperature controlled laboratory maintained at approximately 24°C.

RESULTS

Series of fill and unload tests were conducted with the newly manufactured bins to determine the effects of wear-in. A large reductions in the angle of wall friction resulting from polishing the surface and coating it with organic substances was observed. The reduction in the wall friction angle with repeated loading cycles (LCs) resulted in a decrease of the static vertical wall load-to-total grain load ratio ($VWL/TGL$) from 54% for the first LC to 25% for the 23rd LC in the cylindrical bin and from 38% to 12% in the conical bin as shown in Fig. 4. A similar effect of the angle of wall friction reduction on the dynamic wall load was observed. The dynamic vertical wall-to-total grain load ratio ($VWL/TGL$) measured immediately after the start of discharge decreased from 58% for the first LC to 33% for the 23rd LC in the cylindrical bin and from 40% to 20% in the conical bin. The angle of wall friction decreased from 24 for the first LC to 9 after 23rd LC and the angle of internal friction determined in the triaxial compression test was 25 [5]. For these values of the angles of wall friction and internal friction, Eqs (2) and (8) predict a decrease in the $VWL/TGL$ ratio from 66% to 29% for the cylindrical bin and from 53% to 10% for the conical bin as shown in Fig. 4. The experimental and theoretically determined values of the $VWL/TGL$ are much closer for the last LC than for the first LC. The calculated value of the $VWL/TGL$ ratio decrease is 16% higher for the conical bin than for the cylindrical bin while
the experimentally determined decrease was nearly the same for both bins. The most probable reason for this discrepancy is the very rapid change in the angle of wall friction during the initial period of frictional contact. Therefore, the real value of the angle of wall friction during the first LC will be smaller than the value measured for the new surface. The decrease of the wall friction angle with the increase in length of the shearing path appears to be an exponential relation and is similar for both bins. The reduction of the angle of wall friction was greater during the initial period of frictional contact and decreased as it approached the equilibrium conditions. This phenomenon was observed by Thompson et al. [10] during experiments with wheat repeatedly passed over a steel surface.

The experimentally determined values of the VWL/TGL ratio for both the static and dynamic conditions are in the range of the calculated values. This indicates that the VWL/TGL ratio, which averages any local dynamic overpressures during discharge, can be reasonably approximated by the active stress state as recommended for only the filling and static conditions [1,7].

A change in the direction of the resultant vertical force against the bin wall was observed in the final stage of discharge in both bins. This effect is attributed to the elastic strain recovery in the static grain forming the inflow funnel on the flat-floor. During unloading, as the grain height decreases, the vertical overburden pressures decreases and allows for elastic strain recovery resulting in an upward friction force [2]. It was found that the maximum upward friction force increased linearly with the static vertical floor pressure, $\sigma_y$, which increased with successive LCs. The decrease in the coefficient of wall friction with successive LCs caused an increase in the vertical pressure and, as a result, an increase in the elastic deformation of the grain in the stagnant zone. Figure 5 presents the maximum upward force as a function of the maximum static vertical pressure on the floor for a grain depth $H/D$ ratio equal to 2.75. It was found that an increase in the maximum upward force with the maximum static vertical pressure increase was significantly ($\alpha=5\%$) higher for the conical bin as compared to the cylindrical bin. This is a result of the difference in the contribution of the normal wall pressure, $\sigma_n$, in the upward force in the case of the conical bin.

CONCLUSIONS

1. The Janssen differential slice method provides a good approximation of the wall and bottom loads in both cylindrical and conical bins if supplied with accurate values of the angles of wall and internal friction. Precise evaluation of the angle of wall friction in the initial period of frictional contact is difficult to accurately determine.

2. Reductions in the angle of wall friction resulting from polishing the surface and coating it with organic substances during the initial period of bin operation results in a significant decrease of the vertical wall load-to-total grain load ratio. The higher the wall inclination angle the higher the reduction in the $VWL/TGL$ ratio with a decrease in the angle of wall friction.

3. The change in the inclination of the bin wall from 0 to $6^\circ$ with respect to the vertical direction significantly decreased all measured and calculated load parameters.
ACKNOWLEDGEMENT

The authors wish to express their appreciation to the College of Agriculture, University of Kentucky, for sponsoring Dr. Horabik's and Molenda's visits to the Biosystems and Agricultural Engineering Department which made the research reported in this article possible.

This work was partly supported by the State Committee for Scientific Research, Poland under the Grant No. 5 P06F 024 10.

REFERENCES