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COMPRESSION OF BERRY-LIKE CELL BETWEEN TWO PLATES - ROLE OF INTERNAL STRESS

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A b s t r a c t. A simple model describing the compression of a round object covered by a membrane and filled to different levels with a liquid (berry-like cell), has been proposed. The radius of contact area is approximately proportional to the relative compression of the model object. For low compression levels the relationship between compression force and the relative compression can be expressed by a power equation. The parameters of the power equation strongly depend on the filling level of the model object. The multiplicative parameter has a total minimum when the filling ratio equals 1 and in the same case a maximum for the exponent (n') of the relation is usually observed. Asymptotic values of the exponent are 2 (for high values of the filling ratio) and 1 (for low values of the filling ratio). The values of the exponent n' given by the model are approximately the same as values experimentally determined for compression of berry-like fruits between two plates.

K e y w o r d s: berry-like cell, compression, internal stress

INTRODUCTION

In a previous paper [2] a simple elastic model of fleshy fruit was proposed. The model object is formed by thin elastic membranes of spherical shape that are fully filled with a liquid without any pressure. The model is used for modelling the relation between the compression force and the compression strain during the object compression between two plates. The obtained results are supported by experimental compression of berry-like fruits between two plates [1-3]. The model can be used for description of mechanical behaviour not only of berry-like fruits but also of soft plant cells with thin cell walls and high content of cellular sap [5]. Then the same model can be used for description of mechanical behaviour of the soft tissues of plant and/or animal origin.

The model object is very similar to a baloon, but instead of gas a liquid is used as the filling fluid. This type of objects will be denoted by the term 'berry-like cell' in this paper and it has the following properties:

- the object has approximately spherical shape
- surface of the object is covered by thin and approximately elastic membrane with different degree of permeability,
- the internal content of the object can be classified as a liquid and its internal structure can be idealized in different manners.

In this paper the berry-like cell is represented by a spherical body, covered by thin elastic membrane (its thickness is omitted in all expressions for model object dimension) of non-permeable origin that is filled by simple liquid without any internal structure. It is theoretically studied during its compression between two plates in relation to the initial internal pressure.

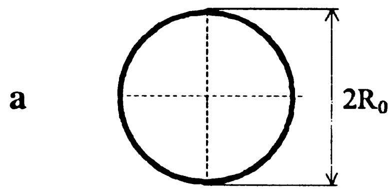
PROBLEM FORMULATION

In Fig. 1 the different filling degrees of the berry-like cell are represented. Full filling of the model membrane by a liquid filler is described by Fig. 1a. The whole internal volume (V_o) of the cell is filled by the filler with volume $V_o^* = V_o$. No initial pressure in the filler can be observed before its compression in this case. The initial radius of the model object is R_o^* and is equal to the radius of the membrane. Initial overfilling of the model object is given in Fig. 1b. The model object has also the same spherical shape as in the case of Fig. 1a, but its volume is higher than in the previous case. This change can be expressed by the filler volume V_o^* and/or in a better way by filling ratio $\nu = V_o^* / V_o$. When compression of the filler can be omitted the radius of the model object is $R_o^* = R_o \nu^{1/3}$. But the overfilled model object has the non-zero initial internal pressure p_{io} and this is why, at least in some cases, the compression of the filler has to be taken into consideration. The last important case is given in Fig. 1c; the berry-like cell is underfilled in this case and has initially barrel form. Its initial height is less than $2R_o$ and typical contact plane circles have non-zero radius R_{so}.

Compression of the berry-like cell between two plates causes formation of some contact areas on its surface, that, under our assumption [2], change the shape of the cell into the barrel-like one (see Fig. 2 and also Fig. 1c). The mutual relation between the most important shape factors, i.e. R and R_s is given by an equilibrium between the internal pressure p_i in the compressed filler:

$$p_i = K \frac{V_o^* - V}{V_o^*} \qquad (1)$$

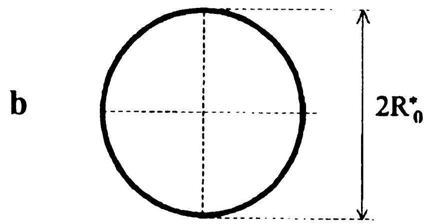
where K is the bulk modulus of the filler and V actual volume of the filler, and p_o is the pressure formed in filler by the membrane tension [2]:



$\nu = V_o^*/V_o = 1$

$p_{io} = 0$

$\xi_{max} = 1$

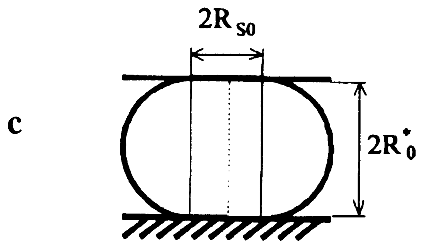


$\nu = V_o^*/V_o > 1$

$R_o^* = R_o \nu^{1/3}$

$p_{io} > 0$

$\xi_{max} = R_o^*/R_o = \nu^{1/3}$



$\nu = V_o^*/V_o < 1$

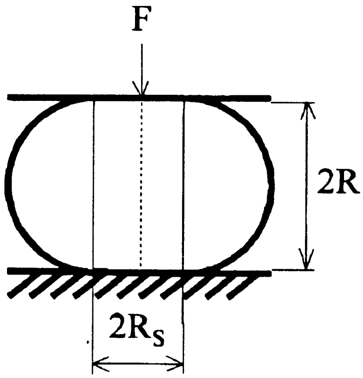
$R_o^* < R_o \nu^{1/3}$

$p_{io} = 0$

$\xi_{max} = R_o^*/R_o = \nu^{1/3}$

$R_{so} > 0$

Fig. 1. Schematic representation of the model object at the beginning of compression for three different filling states. a/ full filling [2], b/ overfilling, c/ underfilling. V_o is the internal volume of a berry-like cell, V_o^* is the initial volume of the model object (it depends on degree of filling), p_{oi} is the initial internal pressure.



$R < R_o^*$

$p_i > 0$

$\xi = R/R_o$

$\xi < \xi_{max}$

Fig. 2. Barrel-like shape of the model object compressed between two plates by axial force F. R_s is the radius of the contact circle.

$$p_o = \frac{2Et}{R}\,\frac{R+R_s}{R+2R_s}\,\frac{S-S_o}{S_o} \qquad (2)$$

where E represents the elastic modulus of the model membrane, t its initial thickness (in undeformed state), S_o the initial area of the membrane (in undeformed state) and S the actual surface area of the berry-like cell.

Deformation force F is balanced by internal pressure p_i on the contact surface:

$$F = \pi R_s^2\, p_i \qquad (3)$$

where instead of p_i from Eq. (1) also p_o from Eq. (2) can be used.

Model theory can be rationalized when some new expressions will be used. There are the above mentioned filling ratio ν, relative actual height of the cell $\xi = R/R_o$, relative radius of the contact area $r_s = R_s/R_o$ and toughness membrane-filler ratio $M = 2\,Et/K\,R_o$. The relative initial cell height $\xi_{max} = R_o^*/R_o$ is also a very useful quantity. It can be used also for the exact definition of relative compression deformation of the model object ε:

$$\varepsilon = 1 - \frac{R}{R_o^*} = 1 - \frac{\xi}{\xi_{max}} \qquad (4)$$

THEORY

The first quantity that has to be determined is the relative initial cell height ξ_{max}. For filling ratios higher or equal 1, the initial shape of the model body is a sphere with R_s equal zero and ξ_{max} is given as a solution of the following equation in the interval $\xi_{max} \in (1,\nu)$:

$$\xi_{max}^4 + M\nu\,\xi_{max}^2 - \nu\,\xi_{max} - M\nu = 0 \qquad (5)$$

that is obtained from equality of equations (1) and (2) and that has trivial solution $\xi_{max} = 1$ for $\nu = 1$. For filling ratios less than 1, ξ_{max} has to be obtained from the barrel-like configuration (see Fig. 1c) with the volume V_o^* and the surface area S_o. These conditions can be rationalized into the following equation in the interval $\xi_{max} \in (\nu/3,1)$:

$$(32 - 3\pi^2)\,\xi_{max}^6 - 3\,(32 - 3\pi^2)\,\xi_{max}^4 -$$
$$2\nu\,(3\pi^2 - 16)\,\xi_{max}^3 + 72\xi_{max}^2 - 48\nu\,\xi_{max} +$$
$$8\nu^2 = 0 \qquad (6)$$

Barrel-like shape of the compressed berry-like cell has a characteristic relation between the relative actual height and the relative radius of the contact area. This relationship created by the necessary balance between the internal pressure (1) and the pressure (2) produced in the filler by membrane tension. This condition can be rewritten into the cubic equation for r_s:

$$2(M\nu + 6\xi^2)\,r_s^3 + 2\xi\,(1+\pi)\,(M\nu + 3\xi^2)\,r_s^2 +$$
$$[2M\nu\,(\pi\xi^2 + 2\xi^2 - 2) + \xi(3\pi\xi^3 + 8\xi^3 - 8\nu)]\,r_s +$$
$$M\nu(\xi^2 - 1) + 4\xi^2\,(\xi^3 - \nu) = 0 \qquad (7)$$

Compression of berry-like cell between two plates can be described by a compression curve as a relation between the fictitious stress at the initial cross-section σ_i and relative compression deformation ε (see Eq. (4)). σ_i can be expressed as a ratio of axial force F (see Eq. (3)) and initial cross section area of the deformed model object $\pi\,(R_{so} + R_o^*/2)^2$:

$$\sigma_i = \frac{p_i\, r_s^2}{\left(r_{so} + \dfrac{\xi_{max}}{2}\right)^2} \qquad (8)$$

In equilibrium the pressures p_i and p_o have the same values and substitution of Eq. (2) into Eq. (8) leads to the final equation:

$$\sigma_i = \frac{2El}{R_o}\, \frac{r_s^2}{(r_s + \dfrac{\xi_{max}}{2})}\, \frac{\xi + r_s}{\xi + 2r_s} \cdot \left(\xi^2 + \frac{\pi\,\xi\, r_s}{2} + \frac{r_s^2}{2} - 1\right) \qquad (9)$$

The structure of Eq. (9) shows that fictitious stress is proportional to Young modulus E, thickness of membrane l, and reciprocal value of the original radius R_o. Only relative values ν, ξ_{max}, and r_s are in more complicated to σ_i.

RESULTS

Numerical solution of the problem is presented in the form of regression lines in dependence on relative compression. Relative radius of a contact area was obtained by numerical solution of Eq. (7) and is presented in the form given by the following equation:

$$r_s = r_{so} + a\varepsilon^n \qquad (10)$$

where r_{so} is the initial value of this parameter (it is zero for greater or equal one) and a and n are the parameters of the relation. Parameters of Eq. (10) are plotted in Fig. 3. It is clear that the Eq. (10) can be classified as being approximately linear due to value n, which is very close to 1, especially for higher values of M and ν. For values higher or equal one, the values of a and n are very stable, especially in dependence on ν. For values of ν lower than one the dependences of a and n on ν are stronger. In every case that has been analyzed the approximation of the numerical results by Eq. (10) is very tight, with correlation coefficients higher than 0.999.

The relationship between force (or fictitious stress) and relative compression cannot be so clearly classified. In Fig. 4a there are some examples of the computed values of the compression forces plotted against relative compression ε in logarithmic scales for different values of filling ratio. These curves have

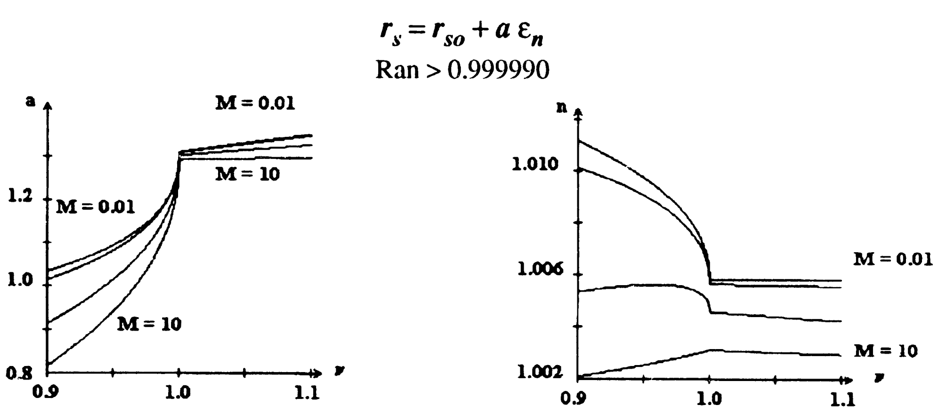


Fig. 3. Parameters of regression Eq. (10) that describes the dependence of the relative radius of a contact area r_s on the relative compression of the object.

approximately linear form, but some deviations can be observed mainly for higher values of relative compression. Linear form corresponds to the power equation, described by the following formula:

$$F = a' \varepsilon^{n'} \qquad (11)$$

where a' and n' are the parameters of the approximation. They must be determined by regression analysis (method of least squares is used for logarithms of F and ε). Correlation coefficient r describes the tightness of the approximation for logarithmic values. Information on the parameters a', b' and r is presented in Fig. 5.

DISCUSSION

The model that is used for quantitative description of the berry-like cell in the course of

M = 10

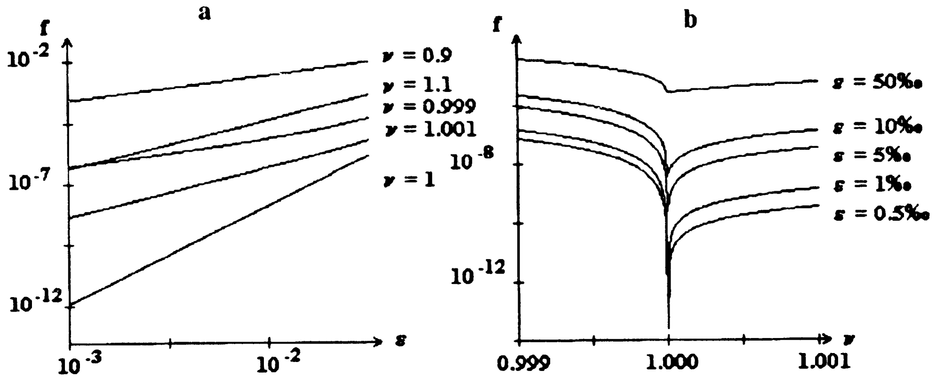


Fig. 4. Examples of compression curves for model object in logarithmic scale for M=10 (a) and the same results plotted in relation to ν (b).

M = 10

$f = a\,\varepsilon^{n'}$

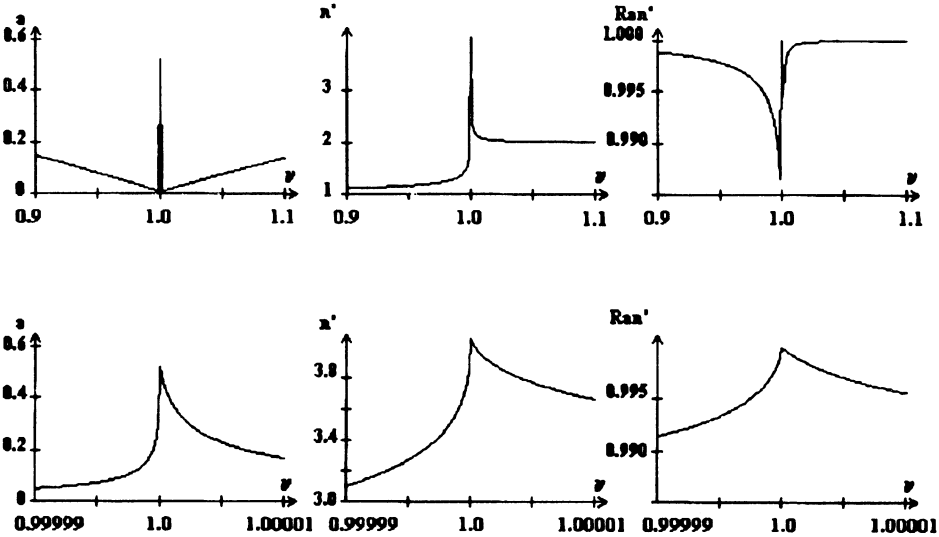


Fig. 5. Regression parameters of Eq. (11) that is used for approximation of the numerical results plotted in relation to ν. Ran' - corresponding correlation coefficient.

its compression between two plates is relatively simple in comparison to the models that have been used for similar purposes [5]. Its predictive ability in the case of contact area seems to be relatively good from the point of view of the relation to relative compression (see Fig. 3) as well as in relation to experiments with latex membranes filled with water (Houška and Blahovec: unpublished results).

The results of computation for the compression force and/or fictitious stress are harder to understand. For small compressions (see Fig. 4b) the compression force decreases with increasing filling ratio up to the value $\nu = 1$, for which the minimum value of the force (relative compression is a constant) is observed. Further increase of the filling ratio is followed by the increasing compression force. For lesser values of the relative compression the much deeper minima of the compression force can be observed. This behaviour has its origin in the different geometry of the berry-like cell for ν higher and lower than 1. For ν more higher than 1 the compression force is controlled mainly by change of contact area between planes and the model objects. This area is approximately a quadratic function of relative compression (see Eq. (10) with $n \sim 1$) and the value of n' in Eq. (11) has to be approximately 2 (see Fig. 5a). When value ν is decreasing to 1, further part of the model is coming into the controlling position, there is a change of internal pressure due to external compression. For ν less than 1 only the meaning of the changes in contact area are much less than for ν higher than 1 and the deformation force is determined mainly by changes of internal pressure in the model object with n approximately equal to 1.

Use of power equation for approximation of compression curve is limited for low values of relative compression (for values lower than 5 % in Figs 4 and 5). Extrapolation of the power relations to higher values of relative compression is strongly limited (see curve for ν=0.999 in Fig. 4a). This behavior is the source of very strange dependence of a' on ν in Fig. 5a; the value a' has a meaning of force F for relative compression ε=1, i.e., for value ε that is physically unrealistic and in every case too high for use of the power approximation. When the relative compression in the formula (11) will be used in percent, parameter a' will depend on ν as the F in Fig. 4b for ε=1 %. It is then clear that parameter a' has a minimum of ν=1 and parameter n' has a maximum of the same value of ν (Fig. 5b). Power approximation of the compression curves is very good for ν=1 and that values of that are much higher then 1 and/or much less than 1 (see Fig. 5c).

The simple model of compressed berry-like cell predicts power relation between compression force and relative compression in form given by Eq. (11), with exponents 1-5. These results are in agreement with our experiments with cherries [4] and berry-like fruits [1].

CONCLUSIONS

Radius of contact area is an approximately linear function of relative compression (n=1 see Fig. 3). Parameters of the power equation are relatively constant for overfilling berry-like cell, with decreasing filling ratio (ν<1) the multiplicative constant a and exponent n' decreases and increases, respectively. For low compression levels the relation between compression force and relative compression can be also expressed by power equation. Parameters of the power equation strongly depend on filling level of the model object. Multiplicative parameter has a total minimum when filling ratio equals 1 and in the same case a maximum for the exponent (n') of the relation is usually observed. Asymptotic values of the exponent are 2 (for high values of the filling ratio) and 1 (for low values of the filling ratio). The values of the exponent n' given by the model are approximately the same as values experimentally determined for compression of berry-like fruits between two plates.

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