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# A SIMPLIFICATION OF EXISTING METHODS FOR CALCULATIONS OF VEGETATION CANOPY AERODYNAMIC CHARACTERISTICS

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A b s t r a c t. Different physical scale models require determination of the vegetation canopy aerodynamic characteristics for calculating the transfer of energy, mass and momentum between the atmosphere and the vegetation-covered surface of the earth. The K-theory inside the canopy was used to obtain a transdental equation for the extinction coefficient, whose solution allows the calculation of zero plane displacement height and roughness length. The proposed procedure is used for calculating the above aerodynamic characteristics for different types of vegetation.

## INTRODUCTION

Different physical scale models (climate simulation, numerical weather prediction, ecosystem simulation) require determination of the fluxes of radiation, water vapour, sensible heat flux and momentum across the lower boundary of the atmosphere. On the other hand, the determination of different fluxes requires special attention to the reason for the sensitivity of the atmosphere to the state of the land surface, particularly in the presence of vegetation [10,11]. The modelling of land surface atmosphere interactions involves several sequential procedures. One of them is the determination of wind speed, shear stress inside and over the vegetation canopy and aerodynamic resistances. For these calculations it is necessary to know the extinction coefficient, roughness, length, zero plane displacement height, momentum transfer coefficient, all of which depend on the vegetation morphology. Numerous authors have devoted attention to the means of calculating the above parameters keeping in mind different length scales and boundary conditions [4,6,7,8].

This paper is focused on simplification of existing methods for calculating the extinction coefficient, zero plane displacement height and roughness length used in different physical scale models. The validity of the proposed method is supported by numerical calculation of the considered aerodynamic characteristics for different types of vegetation.

## THEORY

### Governing equations

The canopy can be represented as a block of porous material of constant density sandwiched between two constant stress layers confined within boundaries $Z_2$, for the height of the canopy top, and $Z_1$, for the height of the canopy bottom.

Under neutral conditions, equations for the transfer of the momentum over and inside the canopy may be written as follows [8]:

Over canopy

$$\tau = \rho \mu_*^2 = \rho \left[ \frac{K\mu}{\ln \frac{Z-d}{Z_o}} \right]^2 \qquad (1)$$

where $\tau$- shear stress, $\rho$ - air density, $\mu_*$ - friction velocity, $K$ - von Karman's constant equal to 0.41, $d$ - zero plane displacement height, $Z_0$ - roughness length.

Inside the canopy ($Z_1 < Z < Z_2$)

$$\frac{\partial \tau}{\partial Z} = \rho \frac{C_d L_d}{P_s} \mu^2 \qquad (2)$$

where $\mu$- wind speed, $C_d$ - leaf drag coefficient, $L_d$ - area-averaged stem and leaf area density, $P_s$ - leaf shelter factor,

$$\tau = \rho K_m \frac{\partial \mu}{\partial z} \qquad (3)$$

where $K_m$ is momentum transfer coefficient.

The derivation of Eqs (1)-(3) can be found in Goudriaan [4]. These equations are commonly used to describe the absorption of momentum by a rough surface. Finally, we define lower boundary conditions [8]:

$$\tau \mid_{Z_1} = \rho C_D \mu^2 \mid_{Z_1} \qquad (4a)$$

and

$$\tau \mid_{Z_2} = \rho K_m \frac{\partial \mu}{\partial Z} \mid_{Z_2} \qquad (4b)$$

where $C_D$ is the drag coefficient; it can be estimated from the size of the roughness elements on the ground.

In order to calculate wind speed and shear stress over and inside the canopy and aerodynamic resistances, the aerodynamic characteristics of canopy $Z_0$ and $d$, are needed. This can be done by solving the system of Eqs (1)-(4) with the five unknowns: $\mu$, $K_m$, $Z_o$, $d$ and $\tau \mid \rho \mid Z_1$. It is well known that the use of the $K$-theory inside the canopy may be physically unrealistic. However, in accordance with Sellers *et al.* [8], it is reasonable to use this method until suitable second-order models can be applied to the problem.

Numerous considerations about the variation of $K_m$ inside the canopy have been offered [2,4,5,8]. The following supposition seems to be the most convenient:

$$K_m = \sigma \mu \qquad (5)$$

where $\sigma$ is a constant [4,8]. In other words some experimental results have indicated that the use of Eq. (5) yields the best results. Moreover, Sellers *et al.* [8] solved the system of Eqs (1)-(4) for a certain set of values $Z_1$, $Z_2$, $L_d$, $C_d$, $P_s$ and $C_D$ and the results obtained were in qualitative agreement with those of Shaw and Pereira [9] obtained with a second-order closure model.

### An approach to the derivation of $\sigma$, $d$ and $Z_0$

First, let us define an expression for wind profile inside the canopy starting from Eqs (2), (3) and (5). Combining these equations we come to

$$\frac{\partial^2 \mu^2}{\partial Z^2} = \beta^2 \mu^2, \qquad (6)$$

where $\beta$, the extinction coefficient, is defined by:

$$\beta^2 = \frac{2C_d L_d}{\sigma P_s} Z_2^2 \qquad (7)$$

then, the Eq. (6) can be solved as

$$\mu = \mu_2 \left[ \frac{Sh \frac{\beta Z}{Z_2}}{Sh \beta} \right]^{1/2} \qquad (8)$$

where $\mu_2$ is the velocity at the top of the canopy [1].

Fundamentally, $Z_o$, $d$ and $\sigma$ depend on the plant geometry and boundary conditions at the interfaces between the flow regime over the canopy and that inside the canopy. The boundary conditions can be derived from the logarithmic wing profile above the canopy on the one hand and the exponential wind profile inside the canopy on the other, to provide continuity in wind speed, momentum transfer coefficient and wind speed gradient.

The first boundary condition is

$$\mu_2 = \Gamma_2 \frac{\mu_x}{K} \ln \frac{Z_2-d}{Z_o} \qquad (9)$$

where $\Gamma_2$ is a constant.

The second boundary condition provides the continuity for $K_m$ and thus

$$K(Z_2-d)\,\mu_* = \sigma\mu_2 \qquad (10)$$

in accordance with Goudriaan [4]. However, Garratt [3] has noted that estimates of momentum transfer coefficient at $Z=Z_2$ were larger than a simple downward extrapolation of Eq. (1) would indicate. In that sense Eq. (10) can be written in the form:

$$\Gamma_1 K(Z_2-d)\,\mu_* = \sigma\mu_2 \,. \qquad (11)$$

The third boundary condition can be defined by:

$$\frac{d\mu}{dZ}/Z_2 = \frac{\mu_*}{K(Z_2-d)} \,. \qquad (12)$$

The list of boundary conditions can be closed with the lower boundary conditions expressed by Eqs (4a) and (4b). Finally, taking into account Eqs (8) and (9), conditions formulated by Eqs (11), (12), (4a) and (4b) can be written as follows:

$$\Gamma_1 K^2 (Z_2-d) = \sigma \ln \frac{Z_2-d}{Z_o} \qquad (13)$$

$$\Gamma_2 (Z_2-d) \ln \frac{Z_2-d}{Z_o} = 2Z_2 \frac{th\beta}{\beta} \qquad (14)$$

$$\sigma = 2C_D Z_2 \frac{tha\beta}{\beta} \qquad (15)$$

where $\alpha$ is defined as $Z_1/Z_2$.

Further, conditions expressed by Eqs (10) and (12) can be linked as:

$$\Gamma_1 K^2 (Z_2-d) = \sigma\Gamma_2 \ln \frac{Z_2-D}{Z_o} \,. \qquad (16)$$

In that manner we have derived a system of the four nonlinear Eqs: (7), (14), (15) and (16) for $\beta, \sigma, Z_o$ and $d$. To slove this system an iterative procedure should be applied that requires specification of the initial conditions and the additional cost of computational time. However, the calculation of $\beta, \sigma, Z_o$ and $d$ can be simplified in the following manner.

Taking into account Eq. (7) and introducing $\chi$

$$\chi = \frac{C_d L_d}{P_s} (1-\alpha) Z_2 \qquad (17)$$

after some manipulation it is simple to obtain the following equation:

$$(1-\alpha)\,\beta tha\beta = \frac{\chi}{C_D} \,. \qquad (18)$$

It is obvious that $\chi$, as introduced by Eq. (17), depends on the plant geometry only [8]. Now, combining Eqs (14) and (16), simple manipulation produces:

$$\left(1-\frac{d}{Z_2}\right)^2 = \frac{4\chi}{K^2(1-\alpha)} \frac{th\beta}{\beta} \qquad (19)$$

i.e., the expression for zero plane displacement height, $d$. In deriving Eq. (19) it was taken that $\Gamma_1 = 1$, as follows from the continuity of shear stress in the upper level of the vegetation canopy. Finally, from Eq. (17), the expression for roughness length becomes:

$$\frac{Z_0}{Z_2} = \left(1-\frac{d}{Z_2}\right) e^{-\frac{2th\beta}{\beta\Gamma_2\left(1-\frac{d}{Z_2}\right)}} \,. \qquad (20)$$

Some experimental results point out that the values of $\Gamma_2$ can be taken in the range 1.5-2.0. In this paper the value of 1.5 is used for $\Gamma_2$.

## NUMERICAL CALCULATIONS

In order to check the validity of the proposed method we have calculated values of the extinction coefficient $\beta$, the zero plane displacement height $d$ and the roughness length $Z_0$ as a function of the leaf drag coefficient, leaf area index and shelter factor.

The extinction coefficient was calculated from Eq. (18). For the value of $\chi$ as defined by Eq. (17) we took into account the values which directly correspond with those used by Shaw and Pereira [9] and Sellers *et. al.* [8]. With regard to $\chi$, the parameter that is hardest to quantify is the shelter factor, $P_s$, reported to be between 1 and 4, depending on vegetation density. The values of $\beta$, obtained by Newton's iterative procedure for different values of $\chi$, are presented in the lower panel of Fig. 1.

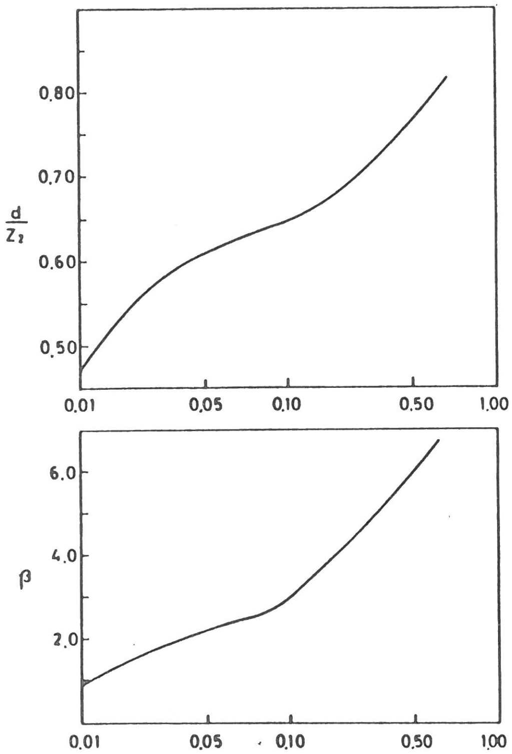


**Fig. 1.** Calculated values of zero plane displacement height, d and extinction coefficient, $\beta$ as a function of plant geometry $\chi$. d is normalized by canopy height $Z_2$.

The calculated values of the extinction coefficient were used to obtain $d$ and $Z_0$ via Eqs (19) and (20), respectively. The values of zero plane displacement height $d$, are shown in the upper panel of Fig. 1. These are in qualitative agreement with those of Shaw and Pereira [9]. The values of the roughness length, calculated via Eq. (20), plotted as functions of 1-$d$ /$Z_2$ and $\chi$, respectively, are shown in Fig. 2. These, too, follow the trend calculated by Shaw and Pereira [9] and Sellers *et al.* [8].

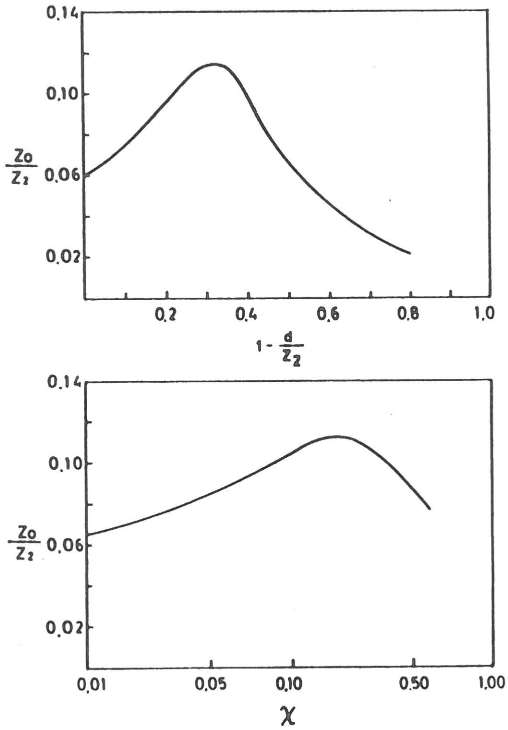


**Fig. 2.** Calculated values of roughness length $Z_0$ plotted against the calculated height between zero plane displacement, d and the canopy top and as a function of plant geometry $\chi$. $Z_0$ is normalized by canopy height $Z_2$.

## CONCLUSIONS

In the above mentioned models, the parameters need to be derived only once. A processor program performs the calculations for a given surface configuration and the values obtained are used as long as the vegetation morphology remains the same.

Finally, let us mention that the proposed procedure for calculating the extinction coefficient, the zero plane displacement heigh, and the roughness length saves considerable computational time.

## REFERENCES


1. **Cowan I.R.**: Mass, heat and momentum exchange between stands of plants and their atmospheric environment. Quart. J. Roy. Meteor. Soc., 94, 523-544, 1968.
2. **Denmead O.T.**: Temperate cereals. In: Vegetation and the Atmosphere (ed. J.L. Monteith). Academic Press, New York, 1-33, 1974.
3. **Garratt J.R.**: Flux profile relations above tall vegetation. Quart. J. Roy. Meteor. Soc., 104, 199-211, 1978.
4. **Goudriaan J.**: Crop Micrometeorology: A Simulation Study. Wageningen Center for Agricultural Publishing and Documentation, Wageningen, 1977.
5. **Jarvis P.G.**: The interpretation of the variations in leaf water potential and stomatal conductance found in canopies in the field. Philos. Trans. Roy. Soc. London, Ser. B., 273, 593-610, 1976.
6. **Mihailović D.T.**: Wind speed and shear stress inside and over vegetation canopy use in GCM. Proc. 20th Int. Conf. on Alpine Meteorology. September 18-25. Sestola, 1988.
7. **Pereira A.R., Shaw R.H.**: A numerical experiment on the mean wind structure inside canopies vegetation. Agric. Meteor., 22, 303-318, 1980.
8. **Sellers P.J., Mintz Y., Sud Y.C., Dalcher A.**: A simple biosphere model (SiB) for use within General Circulation Models. J. Atmos. Sci., 43, 505-531, 1986.
9. **Shaw R.H., Pereira A.R.**: Aerodynamic roughness of a plant canopy: A numerical experiment. Agric. Meteor., 26, 51-65, 1982.
10. **Shukla J., Mintz Y.**: Influence of land-surface evapotranspiration on earth's climate. Science, 215, 1498-1501, 1982.