Int. Agrophysics, 1994, 8, 137-142

NON-LINEAR RHEOLOGICAL METHOD FOR DESCRIBING COMPACTION PROCESSES

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A b s t r a c t. Most compaction processes in agricultural engineering are always associated with increasing elastic moduli showing definitely non-linear character. At the same time, the non-linear viscoelasticity is still less developed and hardly ever used for describing compaction processes.

A new non-linear rheological approach is suggested using varying E-moduli and multiple relaxation times. The E-moduli are expressed as a function of strain and the multiple relaxation times are also dependent on the strain or E-moduli.

In order to verify the proposed method experimental measurements on selected agricultural materials (silage, soil, saw dust etc.) were, undertaken. The measurements and calculation have shown the proposed method is suitable to describe compaction processes in a wide range of pressure as a function of loading velocity.

K e y w o r d s: compaction processes, rheological method, sawdust, corn forage

INTRODUCTION

Compaction processes in agricultural technology play an important role and a lot of machines is working on the compaction principle. The compaction process for bulk agricultural materials is always associated with a strong increase in the modulus of elasticity and the loading velocity has an apparent influence on the stress-strain relationship.

The existing non-linear rheological methods are not suitable to calculate or predict compaction processes for bulk agricultural materials. The linear method in the common form is also not suitable to describe these processes showing a thousend times increase in the modulus of elasticity.

In order to overcome these difficulties a quasi-linear viscoelastic model is suggested. This model supposes varying E-moduli as a function of strain and multiple relaxation and retardation times allowing a more realistic description of the material behaviour for different loading velocities. The proposed model is experimentally verified for selected agricultural bulk materials.

THEORETICAL BACKGROUND

For description of the non-linear viscoelsatic behaviour of compacted materials the extended three-element model was chosen (Fig. 1). Multiple relaxation times and varying

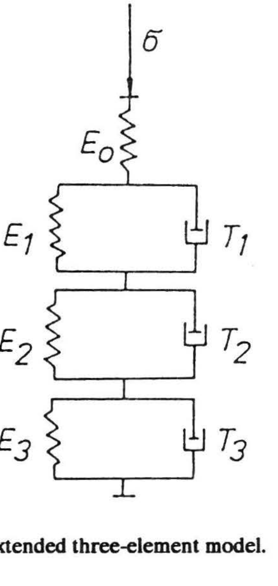

Fig. 1. The extended three-element model.

E-moduli give more possibility to describe the compaction process in better agreement with the experimental observations.

In the model used the spring E_o exhibits elastic response and the Kelvin-models in series represent delayed elasticity with multiple relaxation times.

The non-linear behaviour of material may be taken into account in such a way that the E-moduli will be given and used as a function of strain. The retardation times must be given as a function of strain or elastic modulus. Formulation and use the above functions require special considerations.

The governing differential equation of the model can be solved only for the linear case. In case of a constant deformation rate v_o, the instantaneous strain is:

$$\varepsilon = (v_0 / L)t = a\,t \quad \text{and} \quad d\varepsilon / dt = a$$

Solving the governing differential equation of the model for the linear case, the stress-strain relationship will be:

$$\sigma(t) = E_{oo}\,at + T_1 a\,(E_o - E_{oo1}) \cdot (1 - e^{-t/T_1}) + T_2 a\,(E_{oo1} - E_{oo2}) \cdot (1 - e^{-t/T_2}) + T_3 a\,(E_{oo2} - E_{oo}) \cdot (1 - e^{-t/T_3}) \quad (1)$$

where

$$1/E_{oo} = \sum 1/E_i \; ; \; E_{oo1} = \frac{E_o \cdot E_1}{E_o + E_1} \; ;$$

$$E_{oo2} = \frac{E_o \cdot E_1 \cdot E_2}{E_o E_1 + E_o E_2 + E_1 E_2}.$$

Equation (1) is valid in the time interval $0<t<k_1$, where t_1 denotes the end of the loading period. The subsequent relaxation under constant deformation occurs according to the following expression:

$$\sigma(t) = E_{oo}\,at_1 + T_1 a\,(E_o - E_{oo1}) \cdot (e^{-(t-t^1)/T_1} - e^{-t/T_1}) + T_2 a\,(E_{oo1} - E_{oo2}) \cdot (e^{-(t-t_1)/T_2} - e^{-t/T_2}) + T_3 a (E_{oo2} - E_{oo}) \cdot (e^{-(t-t_1)/T_3} - e^{-t/T_3}) \quad (2)$$

The above equations are normally valid for the linear case with constant E-moduli. As a first approximation, variable E-moduli may also be used, if their functions are correctly defined. The modulus of elasticity for a curved line can generally be defined either the instantaneous modulus ($E=d\sigma/d\varepsilon$) or the secant modulus (Es). The difference in their use may be demonstrated in Hook's law:

$$\sigma = \int E(\varepsilon)\, d\varepsilon \quad \text{and} \quad \sigma = E_S(\varepsilon)\varepsilon$$

Because Eqs (1) and (2) are obtained by integration, therefore, the secant moduli of elasticity must here be used. The same consideration is related to the multiple relaxation or retardation times.

The stress-strain relationship for compaction processes may generally be described with the following equations [1]:

$$\sigma = K \left\{ e^{A\gamma_o \frac{\varepsilon}{1-\varepsilon}} - 1 \right\} \quad (3)$$

where γ_o - the initial volume weight; K, A - constants.

The secant modulus has the following form

$$E_S = K \left\{ e^{A\gamma_o \frac{\varepsilon}{1-\varepsilon}} - 1 \right\} / \varepsilon$$

and

$$E_{So} = K A \gamma_o .$$

A more simple equation may be given as:

$$\sigma = A \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right)^{n} \text{ and } E_S = A\,\frac{\varepsilon^{n-1}}{(1-\varepsilon)^{n}} \qquad (4)$$

The model in Fig. 1. contains four E-moduli, therefore, four constants A_0, A_1, A_2, A_3, and the exponent n must be determined experimentally.

The averaged functions of relaxation times may be given in the form:

$$T_i = \text{const.} / E_{\infty}^{m} \qquad (5)$$

showing decreasing relaxation times with increasing moduli of elasticity.

After the loading period a constant stress may be applied (Fig. 2).

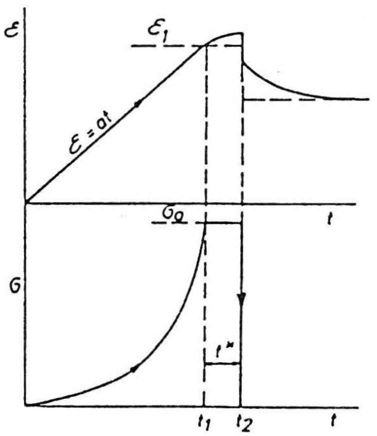

Fig. 2. Creep and rebound of the compressed material.

As a result of creep the deformation will increase as a function of time in the following manner:

$$\varepsilon(t)=\sigma(t_1)/E_{\infty}+(\varepsilon(t_1)-\sigma(t_1)/E_{\infty 1})\cdot e^{-(t-t_1)/T_{r1}}+(\sigma(t_1)/E_{\infty 1}-\sigma(t_1)/E_{\infty 2})\cdot e^{-(t-t_1)/T_{r2}}+(\sigma(t_1)/E_{\infty f2}-\sigma(t_1)/E_{\infty})\cdot e^{-(t-t_1)/T_{r3}} \qquad (6)$$

While the elastic moduli are dependent on the deformation, therefore, Eq. (6) can only be solved by iteration.

The rebound of the material after the time t_2 may be calculated as (simplified form):

$$\varepsilon(t) = \varepsilon(t_2) - (\sigma(t_1)/E_{\infty})\cdot(1-e^{-(t-t_2)/T_r}) \qquad (7)$$

The rebound can also be obtained with the more complex equation with multiple relaxation times similarly to Eq.(6). It should be noted that the rebound will be determined by the instantaneous moduli of elasticity, therefore, in this case these values must be used.

EXPERIMENTAL METHODS

In order to verify the proposed model an experimental work was undertaken with selected agricultural bulk materials. To the experiments corn forage, wet shredded maize, soil and sawdust were used.

The corn forage had a theoretical cut length of 11 mm and different moisture contents were used. The sawdust of beech wood had an average particle size of 0.8 mm. The initial density was 210 kg/m^3.

For slow loading velocities a universal testing machine (INSTRON) was used, for fast loading velocities a special loading device was constructed (Fig. 3) having all measurement units needed.

For confined compression of bulk materials a cylindrical container was used. If the container height is shallow compared to its diameter, the vertical stress varies not too much in the material mass and the material tested can be considered a volume element.

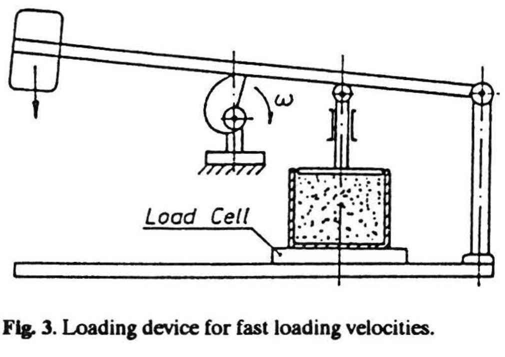

Fig. 3. Loading device for fast loading velocities.

A container of 200 mm dia. was used for the low pressure range, up to 10 bar. For the high pressure range up to 1 000 bar a container of 50 and 60 mm dia. were selected.

The experimental results obtained were processed to determine the E-moduli and the multiple relaxation times as a function of strain for different loading velocities.

An important task was to determine the stress-strain relationship for infinitely high and low loading velocities. Knowing these relationships and those for intermittent loading velocities the viscoelastic behaviour of the material could be obtained.

EXPERIMENTAL RESULTS

Compaction of corn forage

After a given sudden load the relaxation or creep was measured and the results are processed determining the sudden load line (σ_0 versus strain), the multiple relaxation and retardation times and the slow load line (σ_{00} versus strain). Some other finite loading velocities were also measured. Such experimental results for corn forage are summarised in Fig. 4.

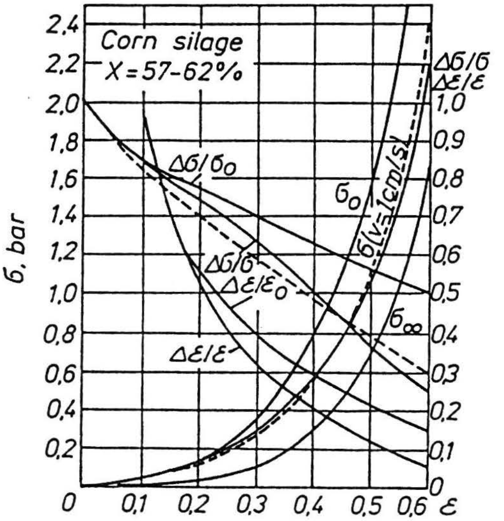

Fig. 4. Stress lines, relative relaxation and creep as a function of strain.

Calculations conducted with the analytical method have shown that the proposed quasi-linear viscoelastic model gives sufficient accuracy for different loading velocities using multiple relaxation times.

The exponent n varies with loading velocities and it differs for the two extreme stress line with a constant value of $\Delta n = 0.3 - 0.4$ independent on material properties. This means that the exponents for elastic response and delayed elasticities are also quite different.

The relative relaxation and relative creep as a function of strain are also plotted in Fig. 4. It should be noted that for compaction processes the decreasing creep and relaxation are characteristic with increasing stress and strain (confined compression).

The multiple relaxation times may be seen in Fig. 5. The exponent m in Eq. (5) is 0.35 and the constants in the equation are 4.5, 35 and 1 000 (they are the intersections for $E_{00}=1$). It is obvious from the figure that the relaxation times decrease with increasing stresses or elastic moduli. The multiple retardation times have the same exponent but the constants are greater (7.5, 45 and 1 200).

The influence of finite loading velocities on the instantaneous stress value can be seen in Fig. 6 using the stress ratio $(\sigma-\sigma_\infty)/(\sigma_0-\sigma_\infty)$.

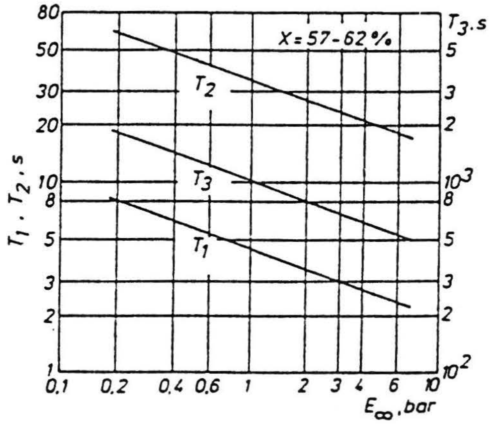

Fig. 5. Multiple relaxation times as a function of asymptotic modulus of elasticity.

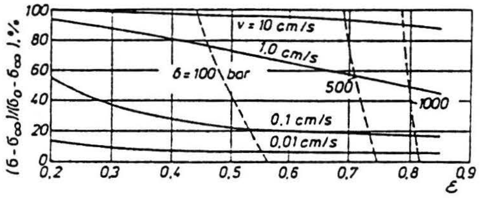

Fig. 6. Influence of loading velocity on the instantaneous stress values as a function of strain.

The constant stress lines are also plotted in the figure. For the tested material 1 m/s loading velocity seems to be nearly as sudden load. It may be seen that the line for constant loading velocity comes nearer to the infinite slow line, when the strain is increasing.

Compaction of sawdust

The pelleting of different agricultural materials requires high pressures. The pressures may be some 1 000 bar or more depending on the required durability.

The stress lines of beech sawdust are summarized in Fig. 7, where the relative stress relaxation and creep are also plotted. For high pressures the two extreme lines come nearer to each other. The relative creep value will be quite small.

The influence of finite loading velocities on the instantaneous pressure is to be seen in Fig. 8 showing also the constant stress lines. The effect of loading velocity is quite similar to those which has been already shown in Fig. 6.

The constants in Eq. (4) are : $A_0=150$, $A_1=160, A_2=450$ and $A_3=700$.

The exponent n for A_0 has a value of 1.5, while for the others $n=2.5$ is valid.

For different end pressures and for several constant stress holding times the rebound of the material was measured. The calculated and measured values are plotted in Fig. 9. From these results may be concluded that the constant stress holding time with increasing pressures has less importance. The calculated and measured values show a good agreement.

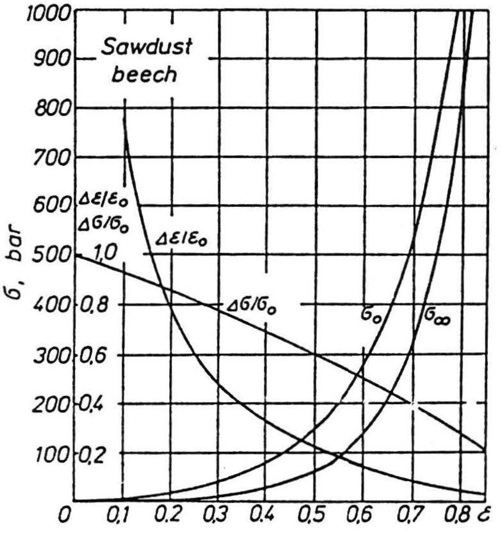

Fig. 7. Compression of sawdust as a function of strain (beech).

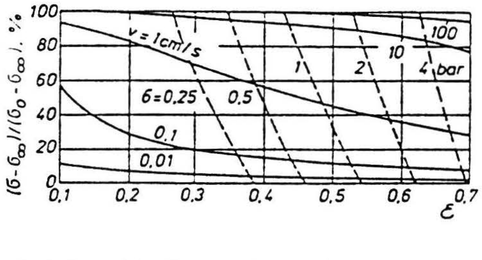

Fig. 8. Effect of loading velocity on the instantaneous pressure as a function of strain.

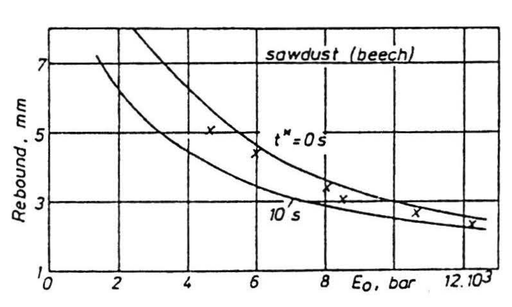

Fig. 9. The rebound of the pressed sawdust.

CONCLUSIONS

Based on theoretical and experimental investigations the following conclusions may be drawn :

1) The confined compression of visco-elastic bulk materials shows special behaviour differing basically from the material behaviour under unconfined loading.

2) In the course of compression the modulus of elasticity and the relaxation times vary as a function of deformation and, therefore, non-linear methods may only be used.

3) The proposed quasi-linear viscoelastic model may be used successfully for different agricultural bulk materials.

4) The quite different materials show a very similar mechanical behaviour under the compaction process.

REFERENCES

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