Int. Agrophysics, 1994, 8, 525-530

YIELD PREDICTION OF VEGETABLE PLANTS USING EXPONENTIAL POLYNOMIAL MODEL (EPM) AND FORECASTS OF TOTAL RAINFALL AND MEAN AIR TEMPERATURE

L. Kuchar

Faculty of Land Reclamation and Improvement, University of Agriculture
Grunwaldzka 53, 50-357 Wrocław, Poland

A b s t r a c t. There has been presented the method of yield forecasting using precipitation amounts and mean air temperature based upon plant phenology and probability distribution of both the random variables. The method enables making forecasts for each recorded phenological stage; the time horizon of the longest of them exceeds 100-110 days. The verification test carried out on the grounds of data coming from different regions of Poland proved a high effectiveness of the method. The use of the model has been illustrated on an example of extreme course of weather according to data obtained from the Experimental Station of the University of Agriculture, Wrocław.

K e y w o r d s: vegetable plants, Exponential Polynomial Model

INTRODUCTION

Early information on the expected course of weather is very useful in many branches of agriculture and related fields. Particularly valuable are long-term forecasts up to two, three months, which make it possible to predict crop yields, plan field works, or make proper use of irrigation systems [2,12,15].

Being able to predict the course of meteorological factors, if only as regards yield prediction, brings serious advantages in such decision-making processes as those concerning the storage and processing of agricultural products, organization of harvest (labour and equipment), the ruling prices, or the export or import in case of surplus or shortage of agricultural products [1,3,10,13,14].

There is a growing interest in models of weather-yield relations, in the forecast of yield-determining weather factors, and in yield predictions, despite real methodological difficulties; the latter may be illustrated by the fact that a majority of research publications deals with the first point, i.e., the description of the relationship between weather and yield [3,11,16].

In the paper a method of forecasting rainfall and mean temperature is shown, which is based on plant phenology and probability distribution of both random variables. This method has been applied for yield forecasting of spring wheat using Exponential Polynomial Model (EPM) - modified version including weights of phenological periods.

Presented method makes it possible to give yield forecast for each recorded phenological stage; the time horizon of the longest of them exceeds 100 to 110 days. Evaluation of method has been made by relative prediction error and absolute error of prediction. A verification test based on data from different regions of Poland proved the method to be very efficient. The method has been illustrated by examples for the data obtained from the Experimental Station of the University of Agriculture in Wrocław.

THEORY

Following method gives possibility to predict yield using weather-crop function and forecast of total precipitation and mean air temperature - two most important variables for many models applications. Both variable are predicted for each recorded phenological stage. In the method, the dates of the phenological stage are given the following notations: x_1 - date of germination; x_i - date of beginning of stage i; x_n - date of full ripeness (n - number of stages).

The random variables are defined: $P^{(i)}$ - total rainfall from 1st December to date x_i ($i=1,...n$); $t^{(i)}$ - mean temperature during period extending from x_1 to x_i ($i=2,...n$), and normally distributed [5,6].

Prediction of weather variables $P^{(k)}$, $t^{(k)}$ is made be vector $\underline{\mu}^*$, using conditional probability distribution in following form:

$$\left(P^{(k)},t^{(k)} \mid P^{(i)} = P^{*(i)}, t^{(i)} = t^{*(i)}, \ldots P^{(1)} = P^{*(1)}\right) \sim N\left(\underline{\mu}^*, \sum{}^*\right) \quad (1)$$

with $i=2, ...n$-1 and $k=i+1,...,n$; (k>i)

where $P^{*(i)}$, $t^{*(i)}$, ...$P^{*(1)}$ are observed values of $P^{(i)}$, $t^{(i)}$, ...$P^{(1)}$; μ^* is the conditional mean vector and $\sum^*$ is the covariance matrix, described in details by Rao [8,9].

Weather-crop function is based on following variables:

$$P = \alpha_1 P^{(1)} + \alpha_2 P^{(2)} + \ldots + \alpha_n P^{(n)}$$
$$t = \beta_2 t^{(2)} + \ldots + \beta_n t^{(n)} \quad (2)$$

where α_1, ..., α_n, β_2, ..., β_n presents weights computed by Monte Carlo methods with restrictions: $\alpha_1 + ... + \alpha_n=1, \beta_2 + ... + \beta_2=1$, ($\alpha_i, \beta_i > 0$); and function from exponential-polynomial class of function, for example [4]:

$$Y(P,t) = \begin{cases} a_0(P-a_1)(t-a_2)\exp(-a_3P-a_4t) & \text{for } P>a_1, t>a_2 \\ 0 & \text{for } P<a_1 \text{ or } t<a_2 \end{cases} \quad (3)$$

where $Y(P,t)$ - estimated yield; a_0, a_1, a_2, a_3, a_4 - coefficients of equation fitted by least square method.

Yield prediction is made using formula [4]:

$$Pr(Y > Y_0) = \int_S \int N^+(\underline{\mu}^*, \sum{}^*)\, ds \quad (4)$$

where y_0 - given yield; $N^+\left(\underline{\mu}^*, \sum^*\right)$ - conditional probability distribution for variable P and t desribed in Eq. 2; S - polygon defined as follows:

$$S = \left\{(P,t) \in R^2 : a_0(P-a_1)(t-a_2)\exp(-a_3P-a_4t) > Y_0\right\}$$

Additionally, expected yield is computed by:

$$EY = Y(P^*, t^*) \quad (5)$$

($\underline{\mu}^* = [P^*, t^*]^T$ in formula (4)).

RESULTS AND DISCUSSION

A preliminary test of the model was done on the basis of data obtained from the Experimental Station of the University of Agriculture in Wrocław. For forecasts the phenological stages of spring wheat have been used. Estimators for formula were calculated on the basis of a period of observation of the spring wheat cultivar Colibri over a period of sixteen years (1971-86), where the following phenological stages were recorded (in parenthesis notation according to Zadoks [17] decimal code): emergence (10), tillering (20), stem elongation (31), head emergence (51), flowering (61), milk ripeness (71), wax ripeness (83), and full ripeness (91).

The preliminary satisfactory results of computations based on empirical data gathered from the Experimental Station of the University of Agriculture, Wrocław, prompted us to verify the method on much broader material. To this end, we used data derived from the network of meteorological stations of the Institute of Meteorology and Water Resource and also from the Cultivar Research Centre at Słupia Wielka.

Rainfall and air temperature values were determined on the basis of standard data derived from synoptic stations. Data on spring wheat phenology of Colibri and

Jara varieties, were taken from sites with soil belonging to the IVth and Vth bonitation classes (thus similar to soil conditions at the Experimental Station).

The number of stages depending on it, the number of forecasts, depends on accessible data and may be different; in the case of the Experimental Station of the University of Agriculture in Wrocław, $n=8$. However, it is often smaller, because all phenological appearances have not always been recorded.

The precision of the method was assessed by relative prediction error E defined as follows [8,15]:

$$E = \frac{\sum_{i=1}^{m} \left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{m} \left(y_i - \bar{y}_i\right)^2} 100\ \% \qquad (6)$$

where y_i is the yield observed; $\hat{y}_i$ is the yield predicted using formula (5); $\bar{y}$ is the average of yield; m is the number of observations.

Table 1. Model error E for the Experimental Station of the University of Agriculture, Wrocław and ten Experimental Stations from various regions of Poland (average values) - spring wheat

Model error	Forecast error ED for stage[a]								
	0	1	2	3	4	5	6	7	8
	Experimental Station of the University of Agriculture, Wrocław								
E=10.4	29.1	25.0	22.7	20.4	18.0	15.6	13.5	12.1	10.9
	Ten Experimental Stations from different regions of Poland								
E=12.7	30.2	25.9	22.6	20.7	18.8	-	-	14.7	

a_0 - before vegetation using mean value; 1 - emergence; 2 - tillering; 3 - shooting; 4 - heading; 5 - flowering; 6 - milk ripeness; 7 - wax maturity; 8 - full ripeness = error of equation fitting.

Table 2. Long-term means, course of weather and forecasts of total rainfall in 1980 - Experimental Station of the University of Agriculture, Wrocław

Stage[a]		1	2	3	4	5	6	7	8
Date		3-05	19-05	25-05	19-06	6-07	24-07	11-08	21-08
Observations $p^*(i)$		195.3	198.6	200.4	218.8	331.5	449.8[b]	511.0[b]	525.5[b]
Means $\bar{p}^{(i)}$		145.1	176.4	197.4	258.2	308.2	341.5	395.0	414.7
Day of formed prediction (stage)			Prediction for stage						
			2	3	4	5	6	7	8
3-05	(1)		214.3	222.1[c]	269.1[c]	332.8	364.2	448.6	453.4
19-05	(2)			209.5[c]	268.4[c]	335.7	384.8	414.1	438.3
25-05	(3)				264.3[c]	325.6	370.1	401.2	429.0
19-06	(4)					288.2[c]	328.0[c]	373.1[c]	405.1[c]
6-07	(5)						383.7	421.1	447.2
24-07	(6)							482.1	511.7
11-08	(7)								530.6

[a]phenological notation see Table 1; [b]different from the mean ($\alpha=0.1$); [c]prediction not accepted ($|P_i - \bar{P}| < |P_i - \hat{P}|$ for notation see formula (6)).

Table 3. Long-term means, course of weather and forecasts of mean air temperature in 1980 - Experimental Station, University of Agriculture, Wrocław

Stage[a]		1	2	3	4	5	6	7	8
Date		3-05	19-05	25-05	19-06	6-07	24-07	11-08	21-08
Observations $t^*(i)$			8.9	9.2	12.9	13.3	13.9	14.8	15.0
Means $\tau(i)$			11.3	12.3	14.0	14.5	14.9	15.5	15.6
Day of formed prediction (stage)			Prediction for stage 2	3	4	5	6	7	8
3-05	(1)	-	11.5[c]	12.4[c]	14.2[c]	14.8[c]	15.1[c]	15.6[c]	16.0[c]
19-05	(2)	-		10.4	12.1	13.5	14.1	15.1	15.1
25-05	(3)	-			12.7	13.5	13.7	15.0	15.0
19-06	(4)	-				13.5	14.0	15.0	15.0
6-07	(5)	-					14.0	14.9	14.9
24-07	(6)	-						15.9	15.0
11-08	(7)	-							15.0

[a]phenological notation see Table 1; [b]different from the mean (α=0.1); [c]prediction not accepted

In the verification, all values of $\hat{y}_i$ were computed by means of the Cross Validation (CV) in its Leave-One-Out (LOO) version [7,9].

Table 1 shows that the errors are monotonous (decreasing horizontally) and suggests that the method can be useful in various agricultural applications.

EXAMPLE

As an example to illustrate the method, it is applied to the 1980 in the Experimental Station of the University of Agriculture in Wrocław, a year with large weather anomalies. Abnormal 1980 was chosen for presentation the method because of difficulties of prediction comparing to normal year. Particularly, following sequence: present course of rainfall and air temperature, reaction of plant (i.e., phenology), forecast of total precipitation, mean air temperature and spring wheat yield is well illustrated. Additionally, deley of yield prediction and for both variables determined by plant reaction is presented too (Fig. 1).

The computations were based on observational series covering a period of 16 years (1971-1986).

1. Forecast for stage 'emergence', after winter period: 3-05

P(Y>2.0 t/ha)=.804 EY=2.91 t/ha
P(Y>2.5 t/ha)=.682
P(Y>3.0 t/ha)=.538 Meteorological factors:
P(Y>3.5 t/ha)=.364 1980 - P=195.3 mm
P(Y>4.0 t/ha)=.198 mean of long period -P=145.1 mm
P(Y>4.5 t/ha) =.036

2. Forecast for stage 'tillering': 19-05

P(Y>2.0 t/ha) =.863 EY=2.68 t/ha
P(Y>2.5 t/ha) =.612
P(Y>3.0 t/ha) =.472 Meteorological factors:
P(Y>3.5 t/ha) =.303 1980 - P=196.8 mm, t=8.9 °C
P(Y>4.0 t/ha) =.169 mean of long
P(Y>4.5 t/ha) =.042 period - P=176.4 mm, t=11.3 °C

3. Forecast for stage 'shooting': 25-05

P(Y>2.0 t/ha) =.908 EY=2.73 t/ha
P(Y>2.5 t/ha) =.697
P(Y>3.0 t/ha) =.447 Meteorological factors:

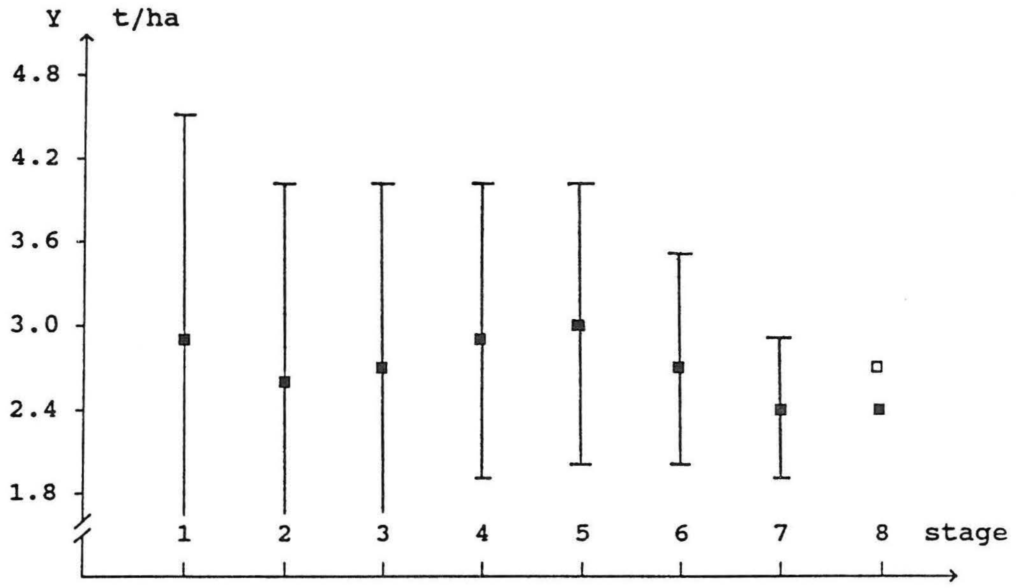

Fig. 1. Successive forecasts of spring wheat yield (■) with 90 % confidence interval and observed yield (□) in 1980, Agricultural and Hydrological Observatory of the University of Agriculture, Wrocław-Swojec.

P(Y>3.5 t/ha) =.328 1980 - P=200.4 mm, t=9.2 °C
P(Y>4.0 t/ha) =.103 mean of long
P(Y>4.5 t/ha) =.011 period - P=197.4 mm, t=12.3 °C

4. Forecast for stage 'heading': 19-06
P(Y>2.0 t/ha) =.936 EY=2.98 t/ha
P(Y>2.5 t/ha) =.742
P(Y>3.0 t/ha) =.486 Meteorological factors:
P(Y>3.5 t/ha) =.363 1980 - P=218.8 mm, t=12.9 °C
P(Y>4.0 t/ha)=.102 mean of long
P(Y>4.5 t/ha)=.063 periods -P=258.2 mm, t=14.0 °C

5. Forecast for stage 'flowering': 6-07
P(Y>2.0 t/ha)=.998 EY=3.02 t/ha
P(Y>2.5 t/ha)=.843
P(Y>3.0 t/ha)=.616 Meteorological factors:
P(Y>3.5 t/ha)=.482 1980 - P=331.5 mm, t=13.3 °C
P(Y>4.0 t/ha)=.210 mean of long
P(Y>4.5 t/ha)=.015 period - P=308.2 mm, t=14.5 °C

6. Forecast for stage 'milk ripeness': 24-07
P(Y>2.0 t/ha) =.933 EY=2.71 t/ha
P(Y>2.5 t/ha) =.667
P(Y>3.0 t/ha) =.494 Meteorological factors:
P(Y>3.5 t/ha) =.183 1980 - P=449.8 mm, t=13.9 °C
P(Y>4.0 t/ha) =.018 mean of long
P(Y>4.5 t/ha) =.001 period - P=341.5 mm, t=14.9 °C

7. Forecast for stage 'wax maturity': 11-08
P(Y>2.0 t/ha) =.972 EY=2.45 t/ha
P(Y>2.5 t/ha) =.563
P(Y>3.0 t/ha) =.204 Meteorological factors:
P(Y>3.5 t/ha) =.001 1980 - P=511.0 mm, t=14.8 °C
P(Y>4.0 t/ha) =.000 mean of long
P(Y>4.5 t/ha) =.000 period -P=395.0 mm, t=15.5 °C

8. Forecast for stage 'full ripeness': 21-08
EY=2.54 t/ha Observed yield Y=2.92 t/ha
mean of long period - 3.17 t/ha.

CONCLUSIONS

The following conclusions can be drawn:

1. The use of conditional probability distribution of rainfall and of mean air temperatures (constructed based on phenological

periods), and weather-crop function from exponential polynomial class of function makes it possible to forecast the values of both variables during the vegetation of the plants.

2. Ample time horizon, even up to 100-110 days, as well as the convergence in time of the precision of forecasts, allows a broad application of the method.

3. Compared with the method of mean value prediction, errors obtained by the application of the present method are reduced by up to 10 per cent.

REFERENCES

1. **Arkin G.F., Maas S.J., Richardson C.W.**: Forecasting grain sorghum yield using simulated weather data and updating techniques. Trans. ASAE, 23, 676-680, 1980.
2. **Bondarenko N.F., Shukowskij E.E.**: Modeling for Productivity of Agroecosystem (in Russian). Gidrometizdat, Leningrad, 1982.
3. **France J., Thornley J.H.M.**: Mathematical Models in Agriculture. A Quantitative Approach to Problems in Agriculture and Related Sciences. Butterworths, London, 1984.
4. **Kuchar L.**: The exponential polynomial model (EPM) of yield forecasting based on meteorological data and phenophase. Agric. For. Meteorol., 46, 339-348, 1989.
5. **Kuchar L.**: Forecasting of total precipitation and mean air temperature during the vegetation period of spring wheat. J. Hung. Meteorol. Serv. Idojaras, 94, 324-329, 1990.
6. **Kuchar L.**: Prediction of meteorological factors during the growth of plants. Zesz. Probl. Post. Nauk Roln., 396, 95-100, 1991.
7. **Michaelson J.**: Cross-validation in statistical climate forecast models. J. Climate Appl. Meteorol., 26, 1589-1600, 1987.
8. **Rao C.R.**: Linear Statistical Inference and Its Applications. John Wiley, Inc., NY, 1973.
9. **Rao C.R.**: Prediction of future observation in growth curve models. Statistical Sci., 2, 434-471, 1987.
10. **Richardson C.W.**: Weather simulation for crop management models. Trans. ASAE, 28, 1602-1606, 1985.
11. **Robertson G.M.**: Weather based mathematical models for estimating development and ripening of crop. Tech. Note No.180, WMO, Geneva, 1983.
12. **Taylor S.E.**: Acceptance and evaluation of weather-related agricultural information. In: Climate and Agriculture Systems Approach to Decision Making (Ed. A. Weiss). Charlestown, South Carolina, 267-279, 1989.
13. **Wisiol K., Hesketh J.D.**: Plant Growth Modeling for Resource Management. Vol. I, II. CRC Press Inc., Boca Raton, Fl.,1987.
14. **De Wit C.T., Penning de Vries F.W.T.**: Predictive models in agricultural production. Phil. Trans. Royal Soc., London, B.310, 309-315, 1985.
15. **World Climate Applications Programme:** Guidelines on Crop Weather Models. WMO-WCP, 50, 1983.
16. **Vaidyanathan A.**: Influence of weather on crop yields. A review of agrometeorologists research. Indian J. Agric. Econ., 35, 127-137, 1980.
17. **Zadoks J.C., Chang T.T., Konzak C.F.**: A decimal code for the growth stage of cereals. Weed Res., Oxford, 14, 415-421, 1974.